ISSN: 2348 - 2117

International Journal of Engineering Technology and Computer Research (IJETCR)

Available Online at www.ijetcr.org

Volume 5; Issue 6; November-December: 2017; Page No. 44-48

Structure and Dynamics of Universe, a contemporary binary operation

Dr. Shobha Lal

Professor of Mathematics and Computing & Dean Faculty of Engineering and Technology,

Jayoti Vidyapeeth Women's University, Jaipur, India

Received 01 Nov. 2017; Accepted 14 Dec. 2017

Abstract

Equation which is a differential equation in R(t) and t represents the basic relationship with which the cosmological models can be constructed. The parameters E and $\wedge$ vare arbitrary each of which may be positive, zero or negative independently of the other. The nature of the models is determined by the actual relationships that hold between R(t) and t which again are dependent on the choice of the values of E and $\wedge$. All the models obtained from Eq. have, however, one feature in common: they all just represent the dynamical picture of the Universe in the classical sense. This is inherent in Eq. which does not contain c, the speed of light, as an entity. But since all our knowledge about the structure and dynamics of the Universe are obtained from light signals, the inclusion of cist relevant in any comprehensive, mathematical formulation of the World models

Key Words: Frame of References, Speed of Light, Cosmology, Contracation, Gravitational Collapse.

Introduction

Suppose that an observer moving with the origin of a system of coordinates observes the physical properties at an arbitrary point A, where OA = r. it can be established that in an expanding Universe in which the cosmological principle strictly holds, the velocity v of the point P relative to the observer O and the density and pressure at P are given by the relations

$v = F(t)\, r$

$\rho = \rho(t)$

$p = p(t)$

It may be noted that under the assumptions we have made, the velocity v is a function of the position of the point observed as well as of the time of observation, while the pressure and density depend only on the time. Equation (21.9) which can be looked upon as the equation of motion of the particle occupying the position P can be integrated t yield

$$r = R(t)\, r_o'$$

Where $r = r_o$ at $t = t_o$, so that $R(t_o) = 1$

and R (t) is a time-dependent scale factor satisfying the relation

$R(t) = F(t)\, R(t)$

R(t) thus represents the expansion parameter and Eq. whows that, if cosmological principle is assumed to hold strictly in the universe, its expansion (or contraction) must be uniform. Equation describe the character of motion of the particle at P. such a motion must also satisfy the conservation laws of mass (equation of continuity) and momentum

The equation of continuity

$\frac{d\rho}{dt} + \rho\nabla.\, v = 0$

Reduces to

$$\frac{d\rho}{dt} + 3\rho(t)\, F(t) = 0$$

In this case by virtue of Eq. since div r = 3. Using can be integrated to

$$\rho(t)\ \rho\ (t_o) = 1/R^3\ (t)$$

Which states that as the liner dimensions of the Universe increase by a factor R(t), the density diminishes inversely as the cube of this factor. This is in accordance with the properties of the Euclidian space which only is appropriate for Newtonian Cosmology.

The vector form of the equations of motion is

$$\frac{dv}{dt} + \frac{1}{\rho}\nabla p - F = 0$$

Where F is the gravitational force per unit mass. With Eqs. and using poison's equation

$$\nabla.F = -4\pi G\rho(t)$$

And taking the divergence of Eq., we get finally

$$3[\frac{d^2F(t)}{dt^2}\ F^2(t)] = 4\pi G\rho(t)$$

G being constant of gravitation.

Eliminating F(t) and its derivative and also r(t) between Eqs. and we have.

$$R^2(t)\ R(t) + \frac{4}{3}\,4\pi G\rho(t_o) = 0$$

For static universe, R(t0 = R(t_o) = 1 and the only solution yielded by us the trivial one ρ(t_o) = 0 that is, the case of an empty Universe. In order to render the Newtonian Cosmology more general so as to be competitive with the Relativistic Cosmology, an additional term has been arbitrarily added to the right-hand side of Poisson's equation, which can therefore be represented as

$$\nabla.F = 4\pi G\rho(t) +$$

Whence,

$$F = -\frac{4}{3}\,4\pi G\rho(t)r + \frac{1}{3}\wedge r$$

∧ used here is the analogue of the cosmological constant which is a very important entity in the relativistic theory. ∧ The introduction of actually ushers in a change in the Newtonian law of gravitation whose effect is supposed to be left only at very great distances. With this introduction Eq. becomes

$$R^2\ (t)\ R(t) + \frac{4}{3}\,4\pi G\rho(t_o) - \frac{1}{3}\wedge R^3\ (t) = O$$

which can be integrated to yield

$$\frac{1}{3}\wedge R^3\ (t) + \frac{8}{3}\pi G\,\frac{\rho(t_o)}{R(t)} - R^2\ (t) = E$$

E being the constant of integration, which actually represents the total energy of the moving particle under consideration.

Equation which is a differential equation in R(t) and t represents the basic relationship with which the cosmological models can be constructed. The parameters E and ∧ vare arbitrary each of which may be positive, zero or negative independently of the other. The nature of the models is determined by the actual relationships that hold between R(t) and t which again are dependent on the choice of the values of E and ∧. All the models obtained from Eq. have, however, one feature in common: they all just represent the dynamical picture of the Universe in the classical sense. This is inherent in Eq. which does not contain c, the speed of light, as an entity. But since all our knowledge about the structure and dynamics of the Universe are obtained from light signals, the inclusion of cist relevant in any comprehensive, mathematical formulation of the World models. The limitation of Newtonian cosmology are imposed by the fact that the latter artificially divides the applicability of gravitational and electromagnetic fields into two completely separated mains. so Newtonian dynamical system can predict nothing about the nature of propagation of light which is essential for a World model. The theory has therefore been rightly superseded by Einstein's theory of General Relativity which combines gravitation with the propagation of light. Since both gravitation and electomagnetismpaly their respective roles united in the theory of

General Relativity, the latter has proved vastly more comprehensive in the study of cosmological problems. Almost all of the more recent cosmological models have been built on the basis of this theory.

According to the theory of General Relativity, the space has a curvature and any events characterized by four general coordinates in space-time labeled by x^i ,= 0,1,2,3. In such a four-dimension space-time, two neighboring events are separated by an interval ds defined by

$$ds^2 = \sum_{i=0}^{3} \sum_{i=0}^{3} g_{ij}\, dx^i\, dx^i$$

Where g_{ij} are ten functions of the four coordinates,ds^2 is called the line element. The concept of geodesic is important in the present discussion. Geodesies are orbits such that the distance between two points measured along it is a minimum. The line element ds^2 may be negative, positive of zero. Whends$^2 < o$, an observer sees the two events occurring simultaneously at a mutual separation of $(-ds^2)^{1/2}$. The interval of separation in this case is called spacelike. On the other hand. $ds^2 > o$ means the separation of two events by a time like interval. In such cases, the observer can travel from one event to the other. In the absence of electromagnetic signal follow geodesies with $ds^2 = o$

The term g_{ij} defining the line element in Eq. plays an extremely important role in the formulation of the theory of General Relativity. The form of g_{ij} tells us what type of coordinate system has been chosen for the problem under investigation. This is vital,because the nature of curvature of space is determined by the particular system of coordinate chosen. the geometry which enables us to separate these two aspects of information from the particular from of g_{ij} is called Riemannian whose basic structure is contained in Eq. (21.26). with this brief introduction, we present in the following structure is contained in Eq. (21.26). which this brief introduction, we present in the following the basic formulae dealing with Relativistic Cosmology.

Under the assumption of the validity of the cosmological principle, that is, if the Universe is assumed homogeneous and isotropic throughout, then H.P. Robertson and A.G. Walker among others have shown that with a suitable choice of units of length and time the four-dimensional line element ds^2 can written as

$$Ds^2 = dt^2 - R^2(t) \left[\frac{(dx^1)^3 + (dx^2)^2 + (dx^3)^2}{\{1 + \frac{1}{4}k\, [(x^1)^2 + [(x^2)^2 + [(x^3)^2]\}^2} \right]$$

The above expression is known as the Robertson-Walker line element which represents a kinematic model of the Universe. This model is unique but for the arbitrariness of the expansion parameter R(t) and the curvature parameter k. The curvature of the three space is given by t=constant and the constant k which can take the values o, + 1 or -1 determines the sign of the space curvature. For any particular value of t, the curvature of space is the same everywhere. The properties of space as determined by particular values of k are as follows.

1. The space is Euclidian whenk = , in such space the surface of a sphere of radius r is $4\pi r^2$ and its volume is $4/3\, \pi\, r^3$.
2. The space is spherical (or elliptical) when k = +1; in this case the surface of the sphere of radius r is less than $4\pi r^2$. The space is in this case closed and has a finite volume.
3. The space is hyperbolic and open when k = -1. In this case surface of a sphere of radius r is greater than $4\pi r^2$.
4. The Einstein field equations are expressed by

$$R_{ij} - \frac{1}{2} R g_{ij} + \wedge g_{ij} = KT_{IJ}$$

Which relate the ten functions g_{ij} of the Universe with its material content and the boundary conditions of the problem. $K = 8\pi G/c^2$ is a constant called the Einstein gravitational constant and $\wedge$ is also a constant called the cosmological constant. If the Robertson-Walker line element is assumed to hold, Einstein field equations in relation (21.28) reduce to the following two equations:

$$\frac{2R}{R} + \frac{R^2}{R^2} + \frac{8\pi Gp}{c^2} = \wedge c^2 - \frac{K}{R^2}$$

$$\frac{R^2}{R^2} - \frac{8\pi Gp}{3} = \frac{1}{3}\wedge c^2 - \frac{K}{R^2}$$

It should be noted that under the assumptions made here only two of the ten field equations contained en Eq. survive. This is because among the terms in the tensor T_{ij} containing the density, pressure of matter, momentum and energy, only those representing the material density $\rho(t)$ and an isotropic pressure p(t) – both of which are functions of time, are relevant in the case. only those solutions of Eqs. (21.29) and(21.30) are physically meaningful which yield r(t) > o and p(t) $\geq$ o for a set of value of the parameters k and $\wedge$ and the function R(t).

Solutions to Eqs. and as such involve much complications. The problem, however, is generally made easier by introducing simplifying as assumptions. One obvious simplification is to take pressure p = o in eq. (21.29). p, of course, here includes all kinds of pressures such as the pressure of thermal motion of the gas, that of random motions of stars and galaxies and the radiation pressure as well. Observations however reveal that at the present epoch of the Universe the **pressure** p is very much smaller compared to the magnitude of the density ρ. Thus putting p = o, the above equation can be integrated to yield.

$$\frac{1}{3}\wedge c^2R^2(t) + \frac{8\pi G p R^3(t)}{3c^2 R(t)} - R^2(t) = k$$

Since $\frac{8\pi G p R^3(t)}{3c^2}$ is constant, Eqs. and are identical in form. Thus, the law of variation of the scale factor R(t) with respect to time in both Newtonian Cosmology and Relativistic Cosmology (when in the latter the pressure is neglected is identical. The only difference is that while in the former the constant E on the right-hand side of Eq. Represents the total energy of the particle, in the latter the constant k on the right-hand side of Eq. represents the spatial curvature. Thus, in fact, the relative importance of the pressure and density in the universe actually governs the relative difference between the Newtonianand Relativistic Cosmology models. This very important result was established by Milne and Mc Crea as early as in 1934.

A second simplifying assumption which has been extensively utilized is that the cosmological constant A has been taken equal to zero. This assumption has been used in constructing most of the relativistic cosmological models. Putting $\wedge$= 0 and subtracting Eqns, we get

$$\frac{R}{R} + 4\pi G\left(\frac{p}{c^2} + \frac{\rho}{3}\right) = 0$$

Using the definition of q given eqns. reduces to

$$H^2 = \frac{4\pi G}{3q}\left(\frac{3p}{c^2} + \rho\right)$$

Since H = R/R, and $\wedge$=o, thus combining Eqns ,we can write.

$$\frac{k}{R^2} = \frac{4\pi G}{3q}\left[\rho(2q - 1) - \frac{3p}{c^2}\right]$$

Equation shows that at any given epoch the curvature of space is related to the energy content of the Universe as defined by its total density and pressure. Since at the present epoch, p =o , relations together sield(for the present epoch)

$$\frac{k}{R^2}\ H^2 (2q - 1)$$

And

$$\frac{3qH^2}{4\pi G} = 2 \times 10^{-29}\ q\ \mathrm{gm\ cm^{-3}}$$

With H = 70 km s^{-1} Mpc^{-1}. The last two relations hsow that the universe will be closed if $\geq 1/2$ and $p > 10^{-29}$ gm cm^{-3}.

COMPREHENSIVE UNDERSTANDING OF UNIVERSE

In the preceding sec tion, we have attempted to present to present the basic mathematical concept that underlie the Newtonian Cosmology and the Relativistic Cosmology. We have also mentioned that the Newtonian theory is inadequate in constructing reliable world models, since the former fails to consider in its framework the phenomenon of the propagation of light. Since Einstein's theory of General Relativity visualizes a much more comprehensive understanding Universe in its large-scale perspective, most of the world models that have been proposed by physicist and astronomers during the present century are based on General Relativity. The correctness of this theory has been demonstrated in the verification fo several of the effects predicted by Einstein on the basis of it. One of the predictions made by Einstein was that because of the warped geometry of space produced by the mass of the sun, the perihelion of Mercury orbit will undergo a prograde protrude precession of about 43".16 per century. The actual amount of precession that has since been measured is 43" per century. The agreement is excellent. Secondly, Einstein's prediction that starlight passing close to the limb of the Sun (observable during a total eclipse shold be defected towards the Sun by 1".75. this has also been observed to agree with fair accurae. Thirdly, the General Relativity predicts that a photon traversing through a potential difference should suffer a redshift. For example a photon of energy hv, while traversing through the gravitational potential of the Sun should suffer a redshift of $\frac{GM_{\Theta}}{R_{\Theta}}\frac{hv}{c^2}$ which is equivalentto a Doppler effect of 0.64 km s^{-1}. Although the verification of this gravitational redshift is a very difficult task, such a shift has been formally confirmed by experiments. These and several other observed relativistic effects have proved the correctness of the theory of General Relativity. Two older theories that have been developed for construction of cosmological models are the Steady-state theory proposed by H. Bondi and T.Gold and by Fred Hoyle, and the Scalar-Tensor theory proposed by C. Brans and R.H. dicke. Models of the Cosmos based on these last two there is will be briefly discussed at the end of this section. Before that we shall discuss a few World models which have been contracted on the basis of Einstein's theory of General Relativity. On this theory again. Two fundamentally different types of models of the Universe have been proposed .

References:

1. Maeder, A. Martinet, L. and Tammann, G. (Eds),observational Cosmology, Geneva Observatory, Switzerland, 1978.
2. Narlikar, J., Introduction to Cosmology, Cambridge University Press, Cambridge, 1993
3. Partridge, R.B.,3K: The Cosmic Microwave Background Radiation, Cambridge University Press, Cambridge, 1995.
4. Peebles, P.J.E. Physical Cosmology, Princeton University Press, Princeton, New