

Evaluation of effectiveness of friction damper in rotating seal using macro slip model by HBM

Dipendra Singh¹, V Suresh Babu², J Srinivasan³

¹Department of Mechanical Engineering, VIT- East Jaipur, India

dipendrabhadauriya0@gmail.com

²Department of Mechanical Engineering, NIT Warangal, India

vsbabu_nitw@yahoo.com

³GE Aviation, JFWTC, Bangalore, India

srinivasanj@ge.com

ABSTRACT

The focus of this work is to develop a method using harmonic balance method (HBM) to calculate the effectiveness of friction damper in rotating seal. This is achieved by using macro slip model technique. A scheme is proposed for evaluating effectiveness of friction damper through quality factor (Q) calculation. The variation of friction coefficient, different viscous damping coefficient, different excitation forces and different rotational speed are taken into consideration.

Keywords: Friction dampers, macro slip model, rotating seal, quality factor, harmonic balance method

INTRODUCTION:

Frictional forces arising from the relative motion of two contacting surfaces are a well-known source of energy dissipation. Sometimes friction is considered as an unwanted effect of the design. There are several types of friction, namely, dry friction, fluid friction, skin friction, internal friction. Dry friction resists relative motion of two solid surfaces in contact. The two types of dry friction are "static friction" between nonmoving surfaces and "kinetic friction" (sliding or dynamic friction) between moving surfaces. In applications such as turbo machinery bladed disks and rotating seals, where structural damping is negligible, dry –friction damping has been widely used to reduce the resonant response of the blades so as to limit the occurrence of wear and premature failure. Having dry friction in the system complicates the dynamic analysis of the system due to its nonlinear nature. Two type of friction damper modeling, namely, macro-slip model and micro slip model are used for such studies.

- Macro slip model assumes the entire friction surface as either slipping or stuck. It is an extensively used method due to its mathematical simplicity. Macro-slip models a friction damper as a spring one end of which slips if the force on the spring is greater than a certain value.

- In more realistic micro-slip model, the entire friction surface is modeled as an elastic body, allowing local slipping in the friction element without gross slip. This allows obtaining damping even in the absence of gross slip

The early research in this area includes the study by J. H. Wang and W. K. Chen (1983) proposed a method to investigate the vibration of a blade with friction damper using Harmonic Balance Method (HBM). HBM is a well-known method for studying nonlinear vibration problems. In this work, a multi-term approximation is proposed. He concluded that the steady state response and other related behavior of a frictionally damped blade can be predicted accurately and quickly by an HBM with a multi term approximation.

Menq and Griffin published a series of articles in the mid-1980s related to the forced response of frictionally-damped structures. Their efforts produced an improved partial slip model which allows for spatially-distributed interface response, a feature which contrasts strongly with many previous analyses using point contact models (1986) and (1986) introduce this improved partial slip model and exercise the model on various systems for comparison to experimental results. Muszynska and Jones (1983) use a hysteresis friction model and single-harmonic solutions to examine the blade vibration

problem for both tuned and mistuned bladed disk problems. They allow for both frictional and elastic coupling between adjacent blades, and conclude that the friction coupling itself may be the source of mistuning.

Other work in this area includes the effort of Wang and Shieh (1991) and Wang and Chen (1993), who examines turbine blade vibrations under conditions of velocity-dependent friction coefficient and higher-order HBM approximations. Two key results are demonstrated. First, (1991) shows that velocity-dependent friction may have a significant impact on damper performance predictions for the one-mode structural model similar to that proposed by Griffin. In particular, the resonant frequency shift for both stiff and flexible dampers can be substantially overestimated—especially in the near optimal pre-load range—if velocity dependence is neglected. This emphasizes the prominent role of friction model in predicting damper performance. Furthermore, resonant stress amplitude in the near-optimal preload range is only affected slightly by the inclusion of velocity-dependent friction. A second critical result reveals the strength of nonlinearity in many friction damper problems. (2012) presents a multi-term HBM approximation to the nonlinear solution and shows that in cases of near-optimal preload, the linearized response using a one-term HBM solution is substantially in error as compared to the three-term solution (which itself is very close to the time integration solution for the full nonlinear equations of motion). Their conclusion is two-fold: i) one-term HBM solutions may neglect critical system dynamics (i.e., nonlinearities) for relevant physical situations of near-optimal preload, and ii) three-term HBM solutions seem to provide an acceptable result as compared to time integration solutions.

Other relevant work includes the contributions of the group at Imperial College led by Sanliturk and Ewins. Their efforts have also focused on HBM analyses of bilinear hysteresis elements. Sanliturk, Imregun, and Ewins (1997) express the hysteresis element as a complex stiffness amenable to frequency-domain analysis, with the bilinear hysteresis element parameters fit directly from experimental data. This interpretation allows convenient analysis using the HBM, and the results again reinforce the computational efficiency and accuracy (compared to time integration) of HBM. Later, Sanliturk,

and Ewins (2007) extend the work to consider 2D friction contact and return to the HBM for approximate solution of the governing equations. The HBM solution accurately predicts response amplitude as compared to the time integration solutions.

In addition to above recently W. Chen and X. Deng (2005) explained about structural damping caused by micro slip along frictional interfaces at bolted joint. George Jureaj Stein, Raduz Zahoransky and Peter Mucka (2007) proposed a method for analysis and simulation of a general single degree freedom oscillatory system with idealized linear viscous damper and dry friction. Later E. Chatelet, G. Michon, L. Manin, and G. Jacquet (2007) proposed a method for choosing the most appropriate contact model from several models, by comparing their efficiency for predicting hysteretic behavior in different applications. Christian M. Firrone (2008) has done study performed on the forced response of a mock up system which simulates the flexural behaviour of two turbine blades with an interposed under platform damper. More recently K. Asadi, H. Ahmadian, H. Jalali (2012) proposed A solution procedure to determine stick–slip transition under single-harmonic excitations is derived. The analytical model is verified using experimental vibration test responses performed on a free-frictionally supported beam under lateral loading.

Formulation of problem

The nonlinear forced vibration response equation of motion for seal is

$$[M] [\ddot{X}] + [C] [\dot{X}] + [K] [X] = \{F\} + \{F_n\} \quad (1)$$

Where $[X]$ = generalized displacement vector

$[M]$ = mass matrix

$[C]$ = Damping matrix

$[K]$ = Stiffness matrix

$\{F\}$ = Linear external force vector

$\{F_n\}$ = Non-linear force vector

If only one-mode approximation is used to investigate the response of the seal near a resonance frequency, then a single degree of freedom system as shown in figure 1. The equation of motion of the single mode model can be

$$m\ddot{x} + c\dot{x} + kx = f - f_n \quad (2)$$

Where f = external excitation force

f_n = Friction force due to friction

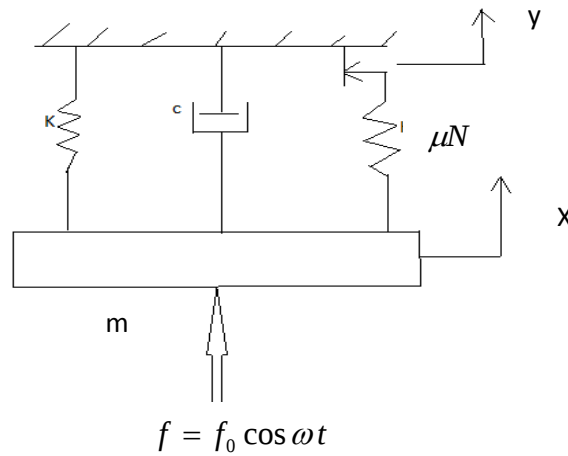


Figure 1: Model of one degree of freedom system [15]

According to macro slip approach, the friction force, f_n , is expressed as

$$\begin{aligned} f_n &= +k_d(x - y) \text{ when } k_d|x - y| \leq \mu N \\ f_n &= +\mu N \text{sign}(\dot{y}) \text{ when } k_d|x - y| \geq \mu N \end{aligned} \quad (3)$$

The external excitation force is a simple harmonic force $f = f_0 \cos \omega t$, the steady state solution of equation (2) is obtained by using Harmonic Balance Method (HBM).

Harmonic balance method

Harmonic balance method (HBM) is a technique to study periodic solution of a nonlinear vibration system. The idea is to expand the periodic solution in Fourier series and to equate the coefficient of the same order to get simultaneous equations using which the solution to the problem is achieved.

One-Term-Approximation: According to Harmonic balance method steady state solution of equation (2) as a Fourier expansion,

$$x = \sum_{i=1}^{\infty} [A_i \cos(i\omega t) + B_i \sin(i\omega t)] \quad (4)$$

For one term approximation

$$\begin{aligned} x &= A_1 \cos(\omega t) + B_1 \sin(\omega t) \\ x &= A_1 \cos \theta + B_1 \sin \theta = R \cos \theta \end{aligned} \quad (5)$$

Where $\theta = \omega t$ and $\theta = \omega t - \phi$

ω = Excitation frequency

ϕ = phase difference between the external excitation force and the response

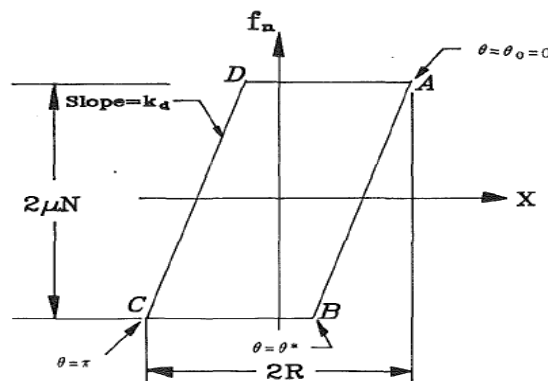


Figure 2: Frictional damping loops, friction force versus the response [15]

During the slip section, the difference between x and y is held constant, which is the horizontal distance between B and middle point of line AB in figure 2

The friction force for one-term approximation is expressed as

$$f_n = k_d(x - y)$$

$$= k_d(x - R) + \mu N, \text{ when } 0 \leq \theta \leq \theta^* \quad (6a)$$

$$f_n = \mu N \text{sign}(\dot{y})$$

$$= \mu N \text{sign}(\dot{x}) = -\mu N, \text{ when } \theta^* \leq \theta \leq \pi \quad (6b)$$

Equation (6a) should be equal to equation (6b), when $\theta = \theta^*$, then

$$\theta^* = \cos^{-1}[1 - 2\mu N / (k_d R)] \quad (7)$$

The response is a simple harmonic form, the response cycle divided in to two symmetric half cycles, i.e. the half cycle ABC is equal to the other half cycle CDA in figure 2

$$\text{So, } f_n(\theta + \pi) = -f_n(\theta), \quad 0 \leq \theta \leq \pi \quad (8)$$

Friction force expanded by Fourier series, only first term is kept,

$$f_n(\theta) = F_c \cos(\theta) + F_s \sin(\theta) \quad (9)$$

Where,

$$F_c = (-k_d R / \pi)[\theta^* - 0.5 \sin(2\theta^*)]$$

$$F_s = (-4\mu N / \pi)[1 - \mu N / (k_d R)]$$

Substituting equation (5) and (9) into equation (2) and separate the coefficient of $\cos \theta$ and $\sin \theta$

$$(k - mw^2)R + F_c = f_0 \cos \phi \quad (10a)$$

$$cwR - F_s = f_0 \sin \phi \quad (10b)$$

$$[(k - mw^2)R + F_c]^2 + [cwR - F_s]^2 = f_0^2 \quad (10c)$$

$$\phi = \sin^{-1}[(cwR - F_s) / f_0] \quad (10d)$$

Equations (10c) and (10d) are non-linear equations with two unknowns R and ϕ , which is solved by the traditional Newton-Raphson method. Only the unknown R needs to be found by iteration; the angle ϕ is obtained directly from equation (10d) providing the R is known. The response is then calculated from equation (5).

Two- term Approximation: The steady state response of the system in figure 2 contains only the odd harmonic components of the external excitation force. Because, the response is a harmonic function, the response cycle can be divided in to two symmetric half cycles as shown in figure 2, Therefore, the response for the two-term approximation,

$$x = A_1 \cos(wt) + B_1 \sin(wt) + A_3 \cos(3wt) + B_3 \sin(3wt)$$

$$x = A_1 \cos \underline{\theta} + B_1 \sin \underline{\theta} + A_3 \cos(3\underline{\theta}) + B_3 \sin(3\underline{\theta}) \quad (11)$$

Let the maximum value of x be denoted by A_m and the corresponding $\underline{\theta}$ be denoted by $\underline{\theta}_0$.

The friction force during the stick

$$f_n = k_d(x - A_m) + \mu N$$

$$\text{portion, } = k_d[A_1(\cos \underline{\theta} - \cos \underline{\theta}_0) + B_1(\sin \underline{\theta} - \sin \underline{\theta}_0) + A_3(\cos 3\underline{\theta} - \cos 3\underline{\theta}_0) + B_3(\sin 3\underline{\theta} - \sin 3\underline{\theta}_0)] + \mu N,$$

$$\text{when } \underline{\theta}_0 \leq \underline{\theta} \leq \underline{\theta}^* \quad (12)$$

During the slip portion, friction force,

$$f_n = -\mu N, \text{ when } \underline{\theta}^* \leq \underline{\theta} \leq \underline{\theta}_0 + \pi \quad (13)$$

With equations of stick and slip portion

$$f_n = F_{c1} \cos \underline{\theta} + F_{s1} \sin \underline{\theta} + F_{c3} \cos 3\underline{\theta} + F_{s3} \sin 3\underline{\theta} \quad (14)$$

The coefficient F_{c1} , F_{c3} , F_{s1} and F_{s3} can be found in the appendix. Substitute equation of friction force and response in equation of motion and setting the coefficient of $\cos \underline{\theta}$, $\sin \underline{\theta}$, $\cos 3\underline{\theta}$ and $\sin 3\underline{\theta}$ is equal to zero,

$$(k - mw^2)A_1 + wcB_1 - f_0 + F_{c1} = 0 \quad (15)$$

$$(k - mw^2)A_3 + 3wcB_3 + F_{c3} = 0 \quad (16)$$

$$(k - mw^2)B_1 - wcA_1 + F_{s1} = 0 \quad (17)$$

$$(k - mw^2)B_3 - 3wcA_3 + F_{s3} = 0 \quad (18)$$

The F_{c1} , F_{c3} , F_{s1} and F_{s3} are functions of $\underline{\theta}_0$ and $\underline{\theta}^*$ which are unknown, the four equations (15)-(18) have six unknown. So someone must find other equations to supplement equation (15) to (18). The response x is maximum (value A_m) when $\underline{\theta} = \underline{\theta}_0$. Thus, the following equation should be satisfied

$$\text{velocity} = \partial x / \partial t \text{ at } \underline{\theta} = \underline{\theta}_0 = -A_1 \sin \underline{\theta}_0 + B_1 \cos \underline{\theta}_0 - 3A_3 \sin \underline{\theta}_0 + 3B_3 \cos 3\underline{\theta}_0 = 0 \quad (19)$$

$$\text{acceleration} = \partial^2 x / \partial t^2 \text{ at } \underline{\theta} = \underline{\theta}_0 = -A_1 \cos \underline{\theta}_0 - B_1 \sin \underline{\theta}_0 - 9A_3 \cos 3\underline{\theta}_0 - 9B_3 \sin 3\underline{\theta}_0 < 0 \quad (20)$$

When $\underline{\theta} = \underline{\theta}^*$, the friction forces in equation (14) and (15) should be equal, thus or in a compact form,

$$k_d [A_1 (\cos \underline{\theta}^* - \cos \underline{\theta}_0) + B_1 (\sin \underline{\theta}^* - \sin \underline{\theta}_0) + A_3 (\cos 3\underline{\theta}^* - \cos 3\underline{\theta}_0) + B_3 (\sin 3\underline{\theta}^* - \sin 3\underline{\theta}_0)] + \mu N = -\mu N \quad (21)$$

$$G = k_d [A_1 (\cos \underline{\theta}^* - \cos \underline{\theta}_0) + B_1 (\sin \underline{\theta}^* - \sin \underline{\theta}_0) + A_3 (\cos 3\underline{\theta}^* - \cos 3\underline{\theta}_0) + B_3 (\sin 3\underline{\theta}^* - \sin 3\underline{\theta}_0)] + 2\mu N = 0 \quad (22)$$

It is known that the most important step in using the Newton-Raphson iteration method is the guess of the initial values. One term and two term approximation is used to calculate frictional force and displacement value. One term and two term equations are coded in excel VBA to automate the analysis.

Results and discussion

There are several ways of expressing the damping in a structure; one of the most common is by the quality factor (Q). Quality factor is defined in the form of frequency ratio. Quality factor is calculated using frequency response curve as shown in figure 3. Ratio of resonance frequency to frequency bandwidth between half power points is known as quality factor.

$$Q = \omega / \Delta\omega$$

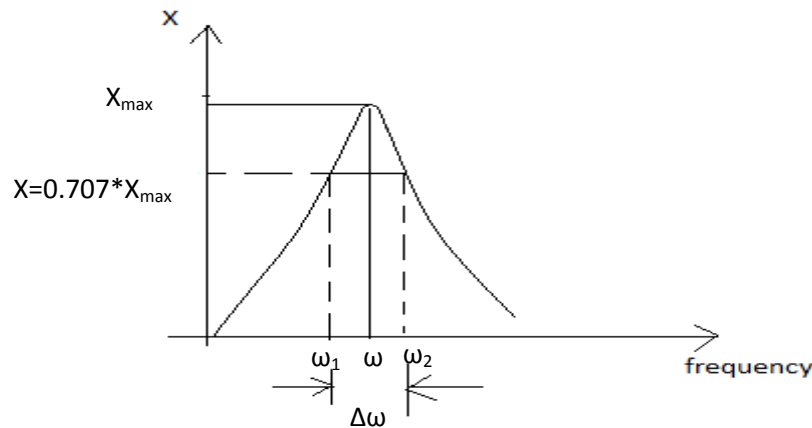


Figure 3: Frequency response curve

$$\Delta\omega = \text{Frequency bandwidth}$$

ω = Resonance frequency or natural frequency

$x_{\max}$ = Maximum amplitude of vibration which occurs at natural frequency with damping

ω_1 and ω_2 = Frequency corresponding to $0.707 * x_{\max}$

Hysteresis loop comparison for seal damper and blade model

Hysteresis curve (i.e. friction force versus displacement), as shown in figure 4, for seal damper model is drawn using excel VBA tool. Seal damper parameters are used to draw the hysteresis curve. For validation purpose hysteresis curve, as shown in figure 5, is drawn for inputs which are given by J. H. Wang and C. K. Chen(1983) for blade model. Area under the hysteresis curve is known as energy of dissipation. Energy dissipation phenomenon explained by I. Lopez and JM Busturia (2004).

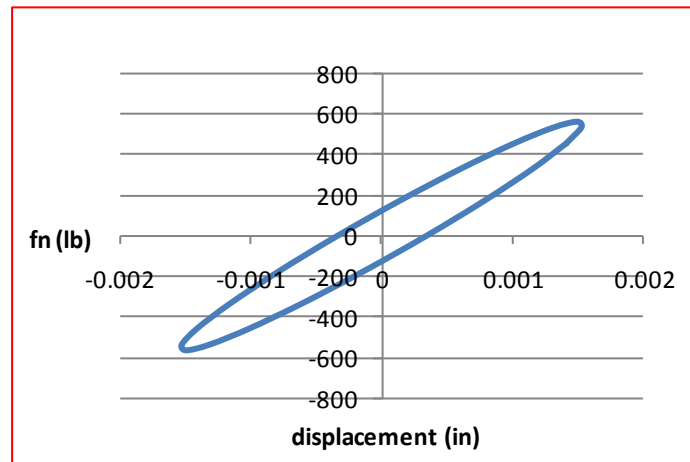


Figure 4: Hysteresis curve for seal damper model

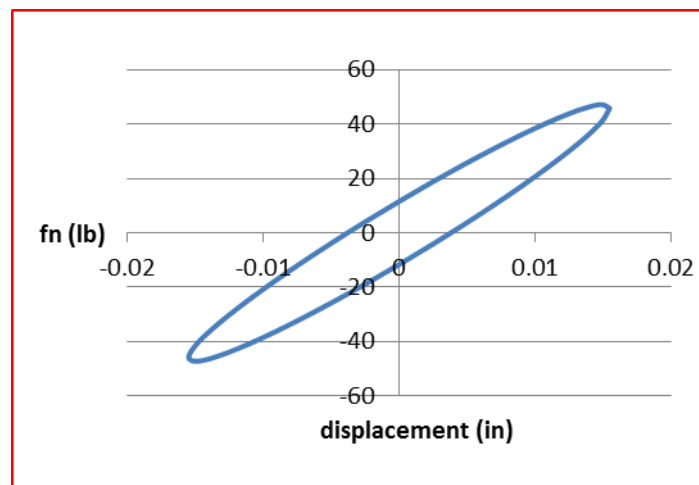


Figure 5: Hysteresis curve for blade model

Hysteresis curves for seal damper model and blade model following same trends. Displacement and frictional force depends on various parameters i.e. mass, stiffness, speed etc., which are different for seal damper and blade model. So numerical value of displacement and frictional force may differ but hysteresis curve for both following same trends.

Frequency response curve for seal damper model

Displacement /response in rotating seal is obtained using inputs e.g. seal stiffness, damper stiffness, mass of seal, mass of damper, shear stress, damper circumferential deflection, natural frequency etc. above inputs are obtained from static and modal analysis of seal damper model. Displacement of seal frequency response curve for seal damper model is obtained using excel tool. Excel tool which is created using equations given by J. H. Wang and W. K. Chen (1983) and

visual basic for application (VBA) language. It is shown in figure 6 expected trend of frequency response curve obtained. As friction coefficient increased response of seal decreases while damping coefficient remains constant. But as frequency ratio reached to one response in seal is maximum. It is shown in figure 7 as damping coefficient increased response of seal decreases while friction coefficient remains constant. It is observed from figure 6 & 7 that frequency response curve follows right trends as found by Balakumar Balachandran in his Vibrations book (2009) & Laetitia Grasser and Herve Mathias, et al. (2007).

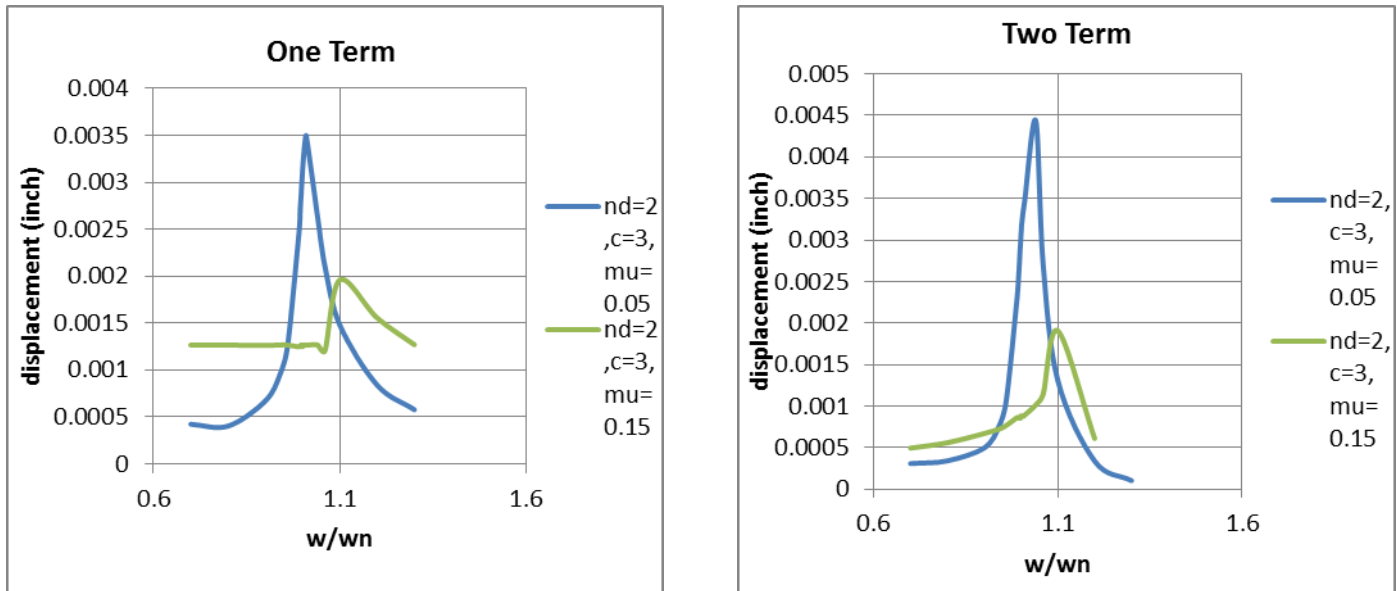


Figure 6: Effect of coefficient of friction on frequency response curve

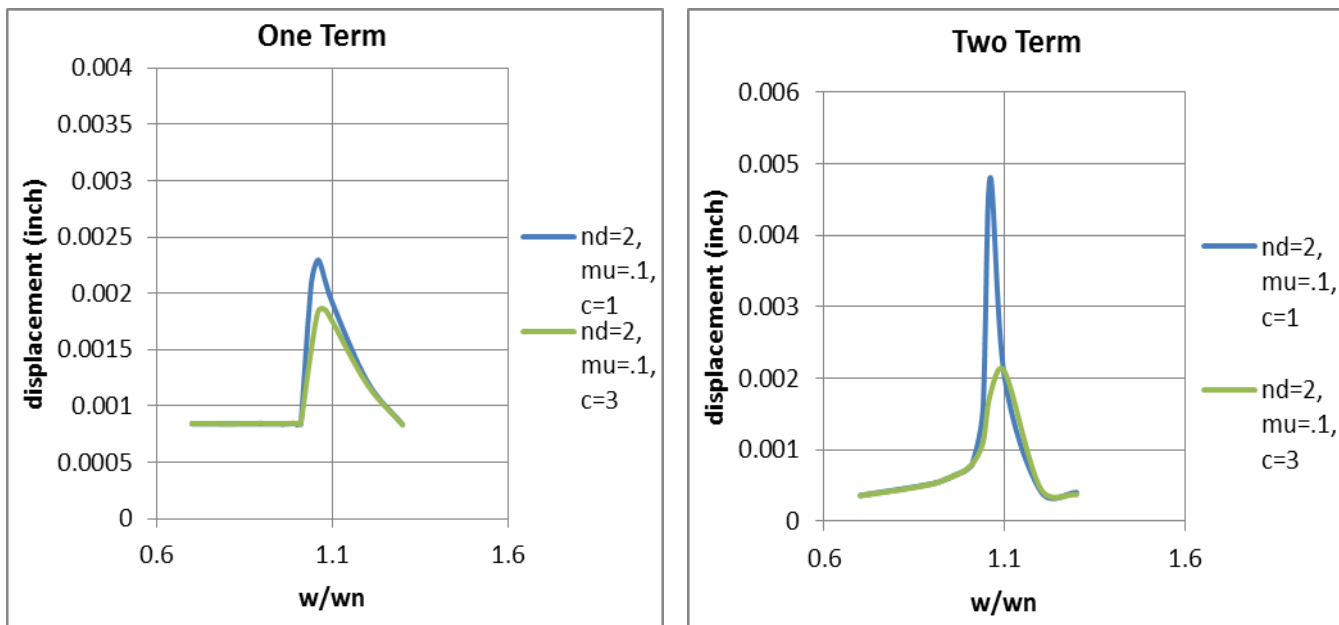


Figure 7: Effect of viscous damping coefficient on frequency response curve

Quality factor (Q) calculation for seal damper model:

Q factor calculated for seal damper model by varying friction coefficient and viscous damping coefficient. It is observed from table 1 & 2 as friction coefficient or damping coefficient increased quality factor decreases and vice versa. So it is concluded that effectiveness of friction damper increased while quality factor decreases.

TABLE 1:

Quality factor at different friction coefficient

ND=2,c=3		
Friction coefficient (Mu)	One Term	Two Term
0.15	5.2704	13.8329
0.05	17.7795	18.5092

TABLE 2

Quality factor at different viscous damping coefficient

ND=2,Mu=0.1		
Viscous damping coefficient (c)	One Term	Two Term
3	6.7999	11.6368
1	8.8517	34.667

Frequency response curve for blade model

To validate the model a frequency response curve as shown in figure 8 is drawn using inputs given by J. H. Wang and W. K. Chen [15]. It is observed from figure 8 that when friction coefficient increases while maximum response of blade decreased. As frequency ratio increased response or displacement in blade increases as shown in figure 8 but as frequency ratio reached to one response in blade is maximum. Frequency response curve follows same trends for blade model as for seal damper model.

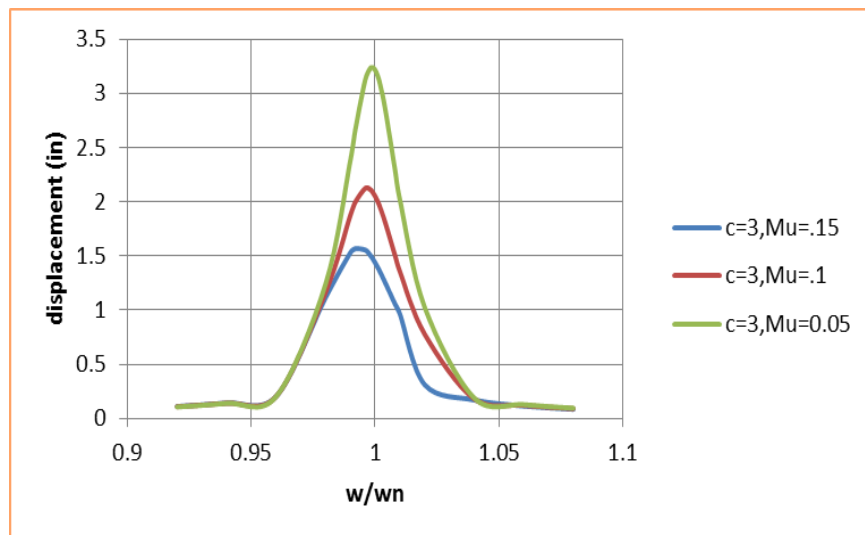


Figure 8: Effect of friction coefficient on frequency response curve

CONCLUSIONS:

Displacement of seal is increased as frequency ratio increases and displacement of seal is maximum for frequency ratio equal to one. Displacement of seal decreases while frequency ratio increased further. Frequency response curve for seal damper model follows right trends. It is clear from frequency response curves that when friction coefficient increased amplitude of vibration of seal decreases because energy of seal dissipated in the form of heat and vibration of seal stabilized at low amplitude. It is observed from above discussion that when damping increased quality factor decreases. In seal damper model quality factor is less than 500. Hence friction damper in rotating seal is effective while quality factor is low.

Acknowledgements:

The author would like to thank the anonymous reviewers for their suggestions and comments in improving the standard of the manuscript.

Notation

C	Viscous damping coefficient
E	Modulus of elasticity
f	Eternal force
f_n	Nonlinear friction force
G	Modulus of rigidity
K	Stiffness of rotating seal
K_d	Stiffness of damper
m	Mass of seal
m_d	Mass of damper
N	Normal load (Centrifugal force)
Q	Quality factor
R	Vibration amplitude
μ	Coefficient of friction
ν	Poisson's ratio
ω	Excitation frequency (rad/sec)
ω_n	Natural frequency (rad/sec)

REFERENCES:

- Asadi K and Ahmadian H et al. (2012) Micro/macro-slip damping in beams with frictional contact interface. *Journal of sound and vibration* 331: 4704-4712.
- Balakumar Balachandran and Edward B Magrab (2009) *Vibrations*. Toronto: CENGAGE learning, pp. 435-535.
- Chatelet E and Michon G et al. (2007) Stick/slip phenomena in dynamics: Choice of contact model, Numerical predictions & experiments. *Mechanism and Machine theory* 43: 1211-1224.
- Chen W and Deng X (2005) structural damping caused by micro slip along frictional interfaces. *International journal of mechanical sciences* 47: 1191-1211.
- Christian M Firrone (2008) dampers with different geometry and comparison with numerical simulation. *Journal of sound and vibration* 323: 313-333.
- George Juraj Stein, Raduz Zahoransky and Peter Mucka (2007) On dry friction modeling and simulation in kinematically excited oscillatory systems. *Journal of sound and vibration* 311: 74-96.
- Laetitia Grasser and Herve Mathias, et al. (2007) Mems Q-factor enhancement using parametric amplification: theoretical study and design of a parametric device. *DTIP of MEMS & MOEMS* ISBN: 978-2-35500-000-3.
- Lopez I and Busturia J M et al. (2004) Energy dissipation of friction damper. *Journal of sound and vibration* 278: 539-561.
- Menq C-H and Bielak J et al. (1986) The influence of micro slip on vibratory response, part i: A new micro slip model. *J. Sound Vib.* 107(2): 279-293.
- Menq C-H and Griffin JH et al. (1986) The influence of micro slip on vibratory response, part ii: A comparison with experimental results. *J. Sound Vib.* 107(2): 295-307
- Morales Ramirez J D and Tirca L (2012) Numerical simulation and design of friction damped steel frame structures. 15 WCEE Lisboa.
- Muszynska A and Jones DIG (1983) A parametric study of dynamic response of a discrete model of turbomachinery bladed disk. *ASME J. Vib., Acoust. Stress, Reliab. Des.* 105: 434-443.
- Sanliturk KY and Ewins DJ (1996) Modeling two-dimensional friction contact and its application using harmonic balance method. *J. Sound Vib.* 193: 511-523.
- Sanliturk KY and Imregun M et al. (1997) Harmonic balance vibration analysis of turbine blades with friction dampers. *ASME J. Vibr. Acoust.* 119: 96-103.
- Seong J Y and Min K W (2011) An analytical approach for design of a structure equipped with friction dampers. *Procedia engineering* 14: 1245-1251.
- Wang JH and Chen WK (1993) Investigation of the vibration of a blade with friction damper by hbm. *ASME J. Eng. Gas Turbines Power* 115: 294-299.
- Wang JH and Shieh WL (1991) The influence of variable friction coefficient on the dynamic behavior of a blade with friction damper. *J. Sound Vib.* 149: 137-145.