ISSN: 2348 - 2117

International Journal of Engineering Technology and Computer Research (IJETCR)
Available Online at www.ijetcr.org
Volume 5; Issue 3; May-June: 2017; Page No. 115-125
Journal Approved by UGC

Periodic Discrete Non-Binary Signals with Perfect Autocorrelation

A. V. Titov[1] and G. J. Kazmierczak[2]

[1] Retired, Mays Landing, USA, Formerly Associate Professor, Electrotechnical University LETI in St. Petersburg, Russia

[2] Cybersecurity Consultant, Princeton, NJ USA

Email: alvastitov@gmail.com

Abstract
This study considers a new approach for designing periodic discrete non-binary signals with perfect autocorrelation properties. When applied in radar systems, these periodic signals provide both high resolution distance measurements and high resolution Doppler speed measurements. The results of this study apply to radar, navigation and communication systems.

Keywords: autocorrelation function (ACF), cross correlation function (CCF), periodic discrete signals, radar and communication systems, Woodward ambiguity function (WAF).

I. Introduction

The autocorrelation function (ACF) is one of the most important signal characteristics used in transmission and information processing systems. Many systems use periodic signals, e.g. radar, sonar, radio navigation systems, communication systems (synchronization and pilot signals), etc. When using periodic signals, it is appropriate to use periodic ACF instead of aperiodic ACF and in this paper we consider only periodic signals with periodic ACF. Many publications consider the properties of periodic ACF of signals; comprehensive lists of references are presented in [1], [2], and [5].

This study considers a new approach for designing periodic discrete signals with perfect ACF [1]. This novel method differs from currently known methods [1], [2], [5], and [12]. Samples of several periodic discrete signals as functions of time are represented in Fig. 1. Aperiodic discrete signals of duration NT consist of N (where N is a positive integer) rectangle pulses with duration T. Periodic discrete signals with a period of repetition NT consist of sequences of aperiodic discrete signals of duration NT; i.e. the period of repetition NT of periodic discrete signals is the same as the length of the aperiodic discrete signals.

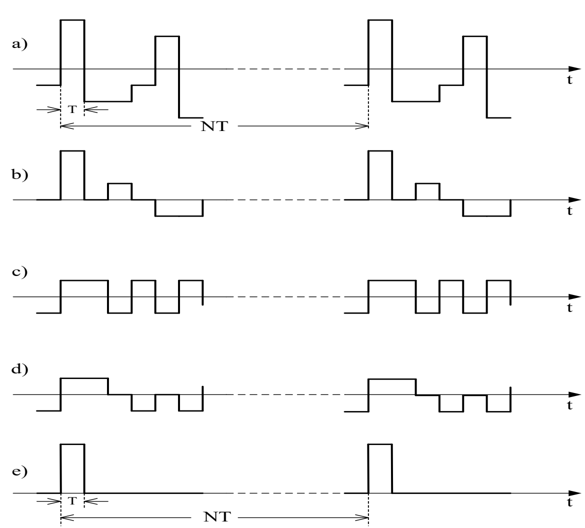

Figure 1

Periodic non-binary signals with no zero values over the full period of repetition are represented in Fig. 1a. Periodic non-binary signals with some zero values are represented in Fig. 1b. Periodic binary signals (+1, -1) and periodic ternary signals (+1, 0, -1) are represented in Fig. 1c and Fig. 1d respectively, and single pulse periodic signals are represented in Fig. 1e. The width of the baseband spectrums of all aperiodic discrete signals of duration NT (Fig. 1) is the same and is defined by the duration of rectangle signal pulse T; i.e. the spectrum width is defined by the main lobe spectrum of the rectangle single pulse of duration T. The same property applies to periodic discrete signals.

Using our new method, it is possible to design non-binary periodic discrete signals with perfect ACF for any signal duration NT (N>2), i.e. for any period of repetition NT. When applying the new method to radar systems, one can create periodic discrete signals capable of both high resolution time delay measurements and high resolution Doppler speed measurements. This paper also considers the use of so called mismatched filters [2], [5], [9], and [10] in order to achieve filter outputs which correspond to periodic signals with perfect ACF.

This study is divided into several sections. Section II presents our new approach for designing periodic non-binary sequences with perfect ACF. In section III, we analyze properties of baseband non-binary signals with perfect ACF, properties of radio signals and the Woodward ambiguity function. In section IV, we consider signal processing using mismatched filters.

II. Periodic Discrete Sequences with Perfect Autocorrelation Function

Periodic discrete signals, as functions of time, consist of a sequence of N rectangle pulses of duration T with a period of repetition NT (Fig. 1).

A numerical finite sequence A_N with length N corresponds to each periodic and aperiodic discrete signal [1],

$$A_N = \{a_i\} = (a_0, a_1, a_2, \ldots, a_{N-1}), \tag{1}$$

where a_i are elements of the sequence, N is the total number of elements, and i = 0,1,2, …, N-1. The value of a_i defines the magnitude of rectangle pulses. Our goal is to find sequences (1) which correspond to signals with perfect ACF. ACF for a periodic signal is also a periodic function with a repetition interval of NT, thus the perfect ACF for periodic signals must have only a single maximum on the interval NT with zero valued side lobes [1],[2], and [5].

Therefore we search for sequences (1) which correspond to signals with perfect periodic ACF. In our search, we chose to use the $A^{(\alpha)}_N$ class of sequences,

$$A^{(\alpha)}_N = \{a^{(\alpha)}_i\} = (a^{(\alpha)}_0, a^{(\alpha)}_1, a^{(\alpha)}_2, \ldots, a^{(\alpha)}_{N-1}) = ((N-\alpha), -\alpha, -\alpha, -\alpha, \ldots, -\alpha), \tag{2}$$

where N is, as above (1), the length of the periodic sequence, i = 0,1,2, …, N-1, $a^{(\alpha)}_i$ are real positive, negative or zero values, and α is the unknown value.

The periodic ACF of sequence (1) can be represented [1] as

$$R_{AN}(\tau) = \sum_i a_i a_{i-\tau(\text{mod } N)}, \quad i, \tau = 0, 1, 2, \ldots, N-1, \tag{3}$$

where i - τ is the mutual shift between sequences $\{a_i\}$ and $\{a_{i-\tau}\}$ in τ steps, which is taken in modulo N(mod N) [1]. Since these sequences are periodic, a modulo N operation means that the mutual shift is the cyclic (or circle) shift of sequences $\{a_i\}$ and $\{a_{i-\tau}\}$. An explanation of the cyclic shift periodic sequences $A_N = \{a_i\}$ is presented in [1].

For sequence $A^{(\alpha)}_N$ in (2), the periodic ACF is

$$R^{(\alpha)}_{AN}(\tau) = \sum_i a^{(\alpha)}_i \, a^{(\alpha)}_{i-\tau(\mathrm{mod}\ N)}, \quad i, \tau = 0, 1, 2, \ldots, N-1. \tag{4}$$

An explanation of the cyclic shift of sequences $\{a^{(\alpha)}_i\}$ and $\{a^{(\alpha)}_{i-\tau}\}$ is presented in Table 1 of the Appendix.

The maximum of ACF (4) for sequence $A^{(\alpha)}_N$ (2) is:

$$R^{(\alpha)}_{AN}(\tau=0) = \sum_i (a^{(\alpha)}_i)^2 = (N-\alpha)^2 + \alpha^2(N-1), \text{ when } \tau = 0. \tag{5}$$

We observe from (4) and the cyclic shifted sequences $A_N^{(\alpha)} = \{ a^{(\alpha)}_i \}$ in Table 1 of the Appendix that the values of ACF $R^{(\alpha)}_{AN}(\tau)$ for any $\tau \neq 0$ on interval N are the same and equal:

$$R^{(\alpha)}_{AN}(\tau\neq 0) = -2\alpha(N-\alpha) + \alpha^2(N-2), \quad \text{for any } \tau\neq 0. \tag{6}$$

In order to achieve a perfect ACF, the values of $R^{(\alpha)}_{AN}(\tau)$ should have a zero value for any $\tau\neq 0$.
Values of α which satisfy this condition for perfect ACF may be found using the following equation:

$$R^{(\alpha)}_{AN}(\tau\neq 0) = -2\alpha(N-\alpha) + \alpha^2 (N-2) = 0. \tag{7}$$

Equation (7) has two solutions for α when $R^{(\alpha)}_{AN}(\tau)$ is zero for any $\tau\neq 0$:

$$\alpha = \alpha_1 = 0 \quad \text{and} \quad \alpha = \alpha_2 = 2. \tag{8}$$

Sequences $A^{(\alpha)}_N$ (2) with α_1 and α_2 can be represented as:

$$\begin{aligned} A^{(\alpha 1)}_N &= ((N-\alpha), -\alpha, -\alpha, \ldots, -\alpha) = (N, 0, 0, 0, \ldots, 0, 0), \text{ if } \alpha = \alpha_1 = 0, \text{ and} \\ A^{(\alpha 2)}_N &= ((N-\alpha), -\alpha, -\alpha, \ldots, -\alpha) = ((N-2), -2, -2, \ldots, -2, -2), \text{if } \alpha = \alpha_2 = 2. \end{aligned} \tag{9}$$

For the case where $\alpha = \alpha_1 = 0$, the periodic sequence $A^{(\alpha 1)}_N$ with length N has only one $a^{(\alpha)}_I$ value which is non-zero and we designate this sequence as $A^{(\alpha 1)}_N = A^{(0)}_N$. Sequence $A^{(0)}_N$ corresponds to the classic case of periodic single pulse signals (Fig. 1d). For the case where $\alpha = \alpha_2 = 2$, the periodic sequence $A^{(\alpha 2)}_N$ does not have any zeros on the entire interval N for any N>2 and it corresponds to periodic non-binary signals (Fig. 1a) with only two values of rectangle pulses: plus (N-2) and minus 2. We designate this sequence as $A^{(\alpha 2)}_N = A^{(2)}_N$.

The ACF of both sequences $A^{(\alpha 1)}_N = A^{(0)}_N$ and $A^{(\alpha 2)}_N = A^{(2)}_N$ are the same (5), (6), and (8):

$$R^{(\alpha)}_{AN}(\tau) = (R^{(\alpha)}_{AN}(\tau = 0), 0, 0, \ldots, 0, 0, 0) = (N^2, 0, 0, 0, \ldots, 0, 0, 0), \tag{10}$$

where:

$R^{(\alpha)}_{AN}(\tau = 0) = (N-\alpha)^2 + \alpha^2(N-1) = N^2-4N+4 +4N-4 = N^2$, if $\alpha = \alpha_2 = 2$,

$R^{(\alpha)}_{AN}(\tau = 0) = N^2$, if $\alpha = \alpha_1 = 0$, and

$R^{(\alpha)}_{AN}(\tau\neq 0) = 0$, if $\alpha = \alpha_1 = 0$ or $\alpha = \alpha_2 = 2$.

Thus, periodic sequences $A^{(\alpha1)}_N = A^{(0)}_N$ and $A^{(\alpha2)}_N = A^{(2)}_N$ (9) on the whole interval N have the same ACF with only one value that is not zero $R^{(\alpha)}_{AN}(\tau)=N^2$, when $\tau = 0$ (10), and $R^{(\alpha)}_{AN}(\tau)=0$ for any $\tau\neq0$. This result means that both periodic sequences $A^{(\alpha1)}_N = A^{(0)}_N$ and $A^{(\alpha2)}_N = A^{(2)}_N$ (9) have the same shape of ACF with perfect periodic autocorrelation properties.

Sequences $A^{(2)}_N$ ($\alpha = \alpha_2 = 2$) (9) which do not have zero values on the interval N can be created for any N>2. For the same value of N (N>2), sequences $A^{(2)}_N$ also include all cyclic shifted and invert polarity sequences. Some examples of cyclic shifted and invert polarity sequences $A^{(2)}_N$ for the same values of N are:

$$\begin{aligned} A^{(2)}_N &= (-2, (N-2), -2, -2, \dots, -2, -2), \\ A^{(2)}_N &= (-2, -2, -2, \dots, -2, (N-2), -2), \\ A^{(2)}_N &= (-(N-2),\ 2,\ 2, \dots,\ 2,\ 2,\ 2), \\ A^{(2)}_N &= (2,\ 2,\ 2, \dots,\ 2, -(N-2),\ 2). \end{aligned} \tag{11}$$

All of the cyclic shifted and invert polarity sequences of (11) have the same periodic perfect ACF (10) for the same value of N (N>2).

The following are examples of sequences $A^{(\alpha2)}_N = A^{(2)}_N$ (9) for partial values of N:

$$\begin{aligned} &N = 3, && A^{(2)}_{N=3} = (1, -2, -2), \\ &N = 4, && A^{(2)}_{N=4} = (2, -2, -2, -2) = 2 * (1, -1, -1, -1), \\ &N = 5, && A^{(2)}_{N=5} = (3, -2, -2, -2, -2), \\ &N = 8, && A^{(2)}_{N=8} = (6, -2, -2, -2, -2, -2, -2, -2) = 2 * (3, -1, -1, -1, -1, -1, -1, -1), \\ &N = 9, && A^{(2)}_{N=9} = (7, -2, -2, -2, -2, -2, -2, -2, -2), \\ &N=12, && A^{(2)}_{N=12} = (10, -2, -2, \dots, -2, -2) = 2 * (5, -1, -1, \dots, -1, -1), \\ &N=16, && A^{(2)}_{N=16} = (14, -2, -2, \dots, -2, -2) = 2 * (7, -1, -1, \dots, -1, -1). \end{aligned} \tag{12}$$

It is important to note that periodic sequences $A^{(\alpha1)}_N = A^{(0)}_N$ and $A^{(\alpha2)}_N = A^{(2)}_N$ (9), (12) are the simplest sequences from all the sequences with perfect autocorrelation properties which can be created utilizing this new approach.

For instance, using the same approach it is possible to show that for the following discrete sequences from the class of $A^{(\alpha)}_N$ sequences (2):

$A^{(\alpha)}_N = A^{(-/+)}_N = ((N-\alpha), \alpha, -\alpha, \alpha, -\alpha, \alpha, -\alpha, \alpha, -\alpha, \alpha, \dots, -\alpha, \alpha)$,

$A^{(\alpha)}_N = A^{(--/++)}_N = ((N-\alpha), -\alpha, \alpha, \alpha, -\alpha, -\alpha, \alpha, \alpha, \dots, -\alpha, -\alpha, \alpha, \alpha)$, and

$A^{(\alpha)}_N = A^{(-+/+-)}_N = ((N-\alpha), \alpha, \alpha, -\alpha, -\alpha, \alpha, \alpha, -\alpha, \dots, -\alpha, \alpha, \alpha, -\alpha)$,

when $\alpha=2$ they can be represented as $A^{(-/+)}_N$, $A^{(--/++)}_N$ and $A^{(-+/+-)}_N$:

$$\begin{aligned} &A^{(-/+)}_N = ((N-2), 2, -2, 2, -2, 2, \dots, -2, 2), \\ &A^{(--/++)}_N = ((N-2), -2, 2, 2, -2, -2, \dots, -2, -2, 2, 2), \text{ and} \\ &A^{(-+/+-)}_N = ((N-2), 2, 2, -2, -2, 2, 2, \dots, -2, 2, 2, -2), \end{aligned} \tag{13}$$

and all of these sequences have perfect periodic ACF.

The first element of these sequences (N-2) (13) is the same as in sequence $A^{(\alpha)}_N$ (2), but the other elements are distinguished by only polarity, i.e. the plus or minus signs are different. In addition, if the sequences $A^{(\alpha2)}_N = A^{(2)}_N$ (9) (12) have perfect periodic ACF for any N (N>2), sequences $A^{(-/+)}_N$ (13) have perfect periodic ACF only for even N (N=4, 6, 8, 10, 12, …) and sequences $A^{(--/++)}_{N}$, $A^{(-+/+-)}_{N}$ (13) have perfect ACF only for N= 4, 8, 12, 16, 20, etc. Some samples of these sequences are presented here:

$$A^{(-/+)}_{N=4} = (2, 2, -2, 2) = 2*(1, 1, -1, 1),$$

$$A^{(--/++)}_{N=4} = (2, -2, 2, 2) = 2*(1, -1, 1, 1),$$

$$A^{(-+/+-)}_{N=4} = (2, 2, 2, -2) = 2*(1, 1, 1, -1),$$

$$A^{(-/+)}_{N=6} = (4, 2, -2, 2, -2, 2) = 2*(2, 1, -1, 1, -1, 1), \tag{14}$$

$$A^{(--/++)}_{N=8} = (6, -2, 2, 2, -2, -2, 2, 2) = 2*(3, -1, 1, 1, -1, -1, 1, 1),$$

$$A^{(-+/+-)}_{N=8} = (6, 2, 2, -2, -2, 2, 2, -2) = 2*(3, 1, 1, -1, -1, 1, 1, -1),$$

$$A^{(-/+)}_{N=12} = (10, 2, -2, 2, -2, 2, -2, 2, -2, 2, -2, 2) = 2*(5, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1).$$

Note that for N=4 the sequences $A^{(2)}_{N=4}$ (12), $A^{(-/+)}_{N=4}$, $A^{(--/++)}_{N=4}$ and $A^{(-+/+-)}_{N=4}$ (14) are identical to sequences of one type of orthogonal Hadamard matrix, namely the orthogonal Hadamard back-circulant matrix for N=4 [3]. The orthogonal back-circulant Hadamard matrix for N=4 can be represented as:

$$H_N = H_{N=4} = \begin{array}{c c} & \downarrow k \\ \begin{bmatrix} + & + & + & - \\ + & + & - & + \\ + & - & + & + \\ + & - & - & - \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array} \tag{15}$$

where k (k= 0, 1, 2, …, N-1) is the matrix row number. The sequences $A^{(-+/+-)}_{N=4}$, $A^{(-/+)}_{N=4}$, $A^{(--/++)}_{N=4}$ (14) and $A^{(2)}_{N=4}$ (12) correspond to sequences of the orthogonal Hadamard back-circulant matrix (15) with row number k =0, k=1, k=2 and k=3 respectively. As it is well known that all periodic sequences of a orthogonal Hadamard back-circulant matrix (15) have perfect periodic ACF [1], [2], and all aperiodic sequences of matrix (15) are also Barker code sequences for N=4 [11].

The following are additional examples of more complicated non-binary sequences with perfect periodic autocorrelation properties which can be created using this new approach:

$$\begin{aligned} &N=9, && A_{N=9} = (1,4,-2,1,-5,-2,1,-5,-2),\\ &N=9, && A_{N=9/0} = 3*(1,2,0,1,-1,0,1,-1,0),\\ &N=12, && A_{N=12} = 3*(1,1,2,1,-1,0,1,1,-2,1,-1,0),\\ &N=16, && A_{N=16} = 2*(1,1,3,-1,1,-3,-1,-1,1,1,-5,-1,1,-3,-1,-1). \end{aligned} \tag{16}$$

These sequences are different from sequences $A^{(2)}_{N=9}$, $A^{(2)}_{N=12}$ and $A^{(2)}_{N=16}$ (12) and sequence $A^{(-/+)}_{N=12}$ (14), however all of them have perfect periodic ACF. Sequences $A_{N=9}$ and $A_{N=16}$ (16) correspond to the type of periodic non-binary discrete signals in Fig. 1a and sequences $A_{N=9/0}$ and $A_{N=12}$ correspond to the type of periodic non-binary discrete signals in Fig. 1b.

In this paper, we do not consider the properties of all of the above sequences with perfect periodic ACF; we only consider the properties of periodic signals which correspond to sequences $A^{(\alpha 1)}_{N} = A^{(0)}_{N}$ and $A^{(\alpha 2)}_{N} = A^{(2)}_{N}$ (9).

III. Periodic Discrete Signals with Perfect Autocorrelation Function

Periodic signals $S^{(0)}_{ANT}(t)$ and $S^{(2)}_{ANT}(t)$,which correspond to sequences $A^{(0)}_N$ and $A^{(2)}_N$ (9), and ACF $R_{SAN}(\tau)$ of these two periodic signals are represented in Fig. 2a, Fig. 2b, and Fig. 2c, respectively.

Periodic signal $S^{(0)}_{ANT}(t)$ (Fig. 2a) is the classic case of a periodic single pulse signal where T is the pulse duration and NT is the period of repetition (where N is a positive integer and usually N>>1) [5], [6]. These signals have only one non-zero value over the entire period of repetition NT (Fig. 1e).

Periodic signals $S^{(2)}_{ANT}(t)$ (Fig. 2b) are the sequences of rectangle pulses of duration T coded in accordance with sequences $A^{(\alpha 2)}_N = A^{(2)}_N$ ($\alpha = \alpha_2 = 2$) (9).

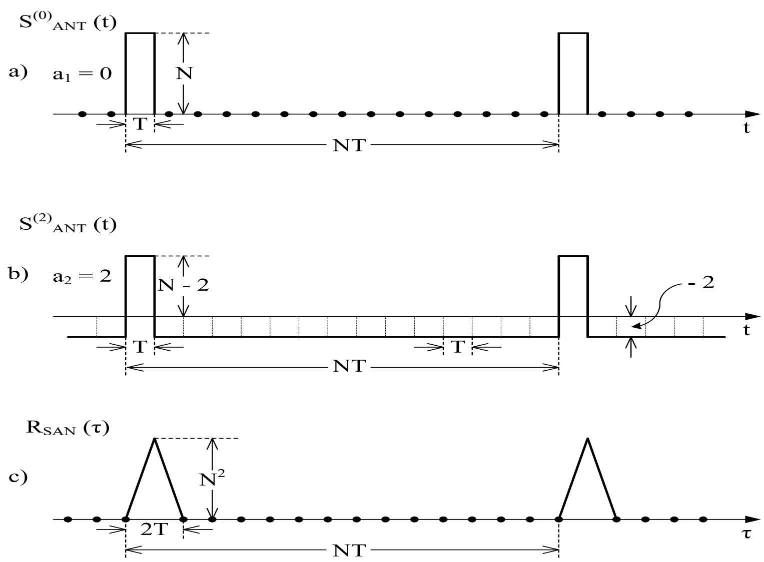

Figure 2

These periodic signals do not have a zero value over the entire period of repetition NT (N>2).

It is necessary to emphasize that periodic signals $S^{(0)}_{ANT}(t)$ and $S^{(2)}_{ANT}(t)$ have the same shape of ACF $R_{SAN}(\tau)$ (Fig. 2c) despite of the differences of their signal structures (Fig. 2a, Fig. 2b). Also, the signals which correspond to sequences $A^{(-/+)}_N$ (N=4,6,8.10, …), $A^{(--/++)}_N$ and $A^{(- +/+-)}_N$ (N=4,8,12, ….) (13), (14) have the same shape of ACF $R_{SAN}(\tau)$ (Fig. 2c). As previously described, the width of the baseband spectrums of both periodic signals $S^{(0)}_{ANT}(t)$ and $S^{(2)}_{ANT}(t)$ and also periodic signals with corresponding sequences $A^{(-/+)}_N$, $A^{(--/++)}_N$, $A^{(-+/+-)}_N$ are the same and is defined by the duration of the rectangle signal pulse T.

The periodic radio signal $S^{(2)}_{RAN}(t)$ coded in accordance with sequence $A^{(2)}_N$ ($\alpha = \alpha_2 = 2$) (9) is represented in Fig. 3c.

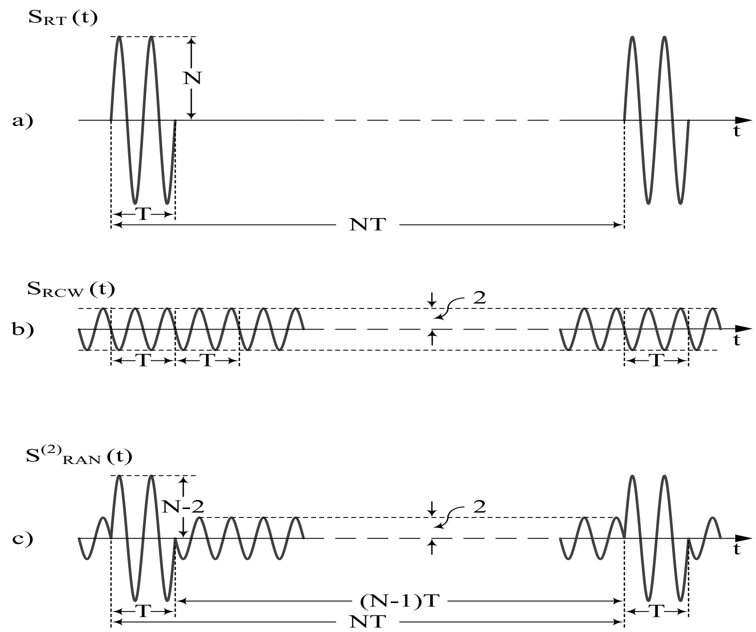

Figure 3

Binary phase coding of carrier frequency is used to code the polarity of sequence $A^{(2)}_N$. The periodic radio signal $S^{(2)}_{RAN}$ (t) with a duration of NT consists of two parts: the first part with a duration of T has zero phase and the second part with a duration of (N-1)T has opposite phase.

Periodic radio signal $S^{(2)}_{RAN}$ (t) can be represented as the sum of two radio signals, namely periodic radio signal S_{RT} (t) of duration T and a period of repetition NT (Fig. 3a), and continuous wave (CW) signal S_{RCW}(t) (Fig. 3b). Notice that these two types of signals are used in different kinds of radar systems, namely in periodic unmodulated single pulse radar systems and in CW radar systems [5], [6].

The Woodward ambiguity function (WAF) is the main characteristic of radar systems [4], [5], [6], [7], and [8]. WAF defines the resolution capability in the time domain and in the frequency domain. In the time domain,WAF defines resolution, i.e. accuracy, of time delay (distance) measurements $\Delta\tau$. In the frequency domain,WAF defines resolution of Doppler shift frequency (speed) measurements $\Delta\omega_D$. For periodic signals, WAF is also a periodic function with a period of repetition NT.

Pulse radars can provide high resolution of time delay measurements $\Delta\tau$=T with prefect ACF defined by the pulse duration T (Fig.3a). But pulse radars can not provide high resolution of Doppler frequency shift (speed) measurements. CW radars can provide high resolution of Doppler shift frequency (speed) measurements $\Delta\omega_D=2\pi/T_M$, where $T_M >> T$ (T_M is the measurement time), but cannot provide high resolution of distance measurements.

The periodic radio signal $S^{(2)}_{RAN}$ (t) (Fig. 3c) combines the features of single pulse signals and CW signals that are beneficial for high resolution measurements in both the time and frequency domains.The periodic radio signal $S^{(2)}_{RAN}$ (t) can provide high resolution of time delay (distance) measurements $\Delta\tau$=T with perfect ACF and can also provide high resolution of Doppler frequency shift (speed) measurements$\Delta\omega_D=2\pi/(N-1)T$during time interval (N-1)T (Fig. 3c - CW signal duration (N-1)T).

Since no perfect WAF has been defined yet for periodic discrete signals, we can introduce the following definition. For periodic discrete signals of duration NT, a perfect WAF on interval NT must have the same shape as the WAF for a periodic single unmodulated pulse signal with a time delay measurement resolution $\Delta\tau$=T (perfect ACF) and with a Doppler frequency shift measurement resolution (potential) $\Delta\omega_D=2\pi/NT$. Notice that T and N are the primary parameters of discrete periodic signals. In accordance with this definition, we observe that periodic signal $S^{(2)}_{RAN}(t)$ (Fig. 3c) provides not only perfect ACF, but also can provide nearly perfect WAF with time delay resolution measurements $\Delta\tau=T$ and Doppler frequency shift resolution measurements $\Delta\omega_D=2\pi/(N-1)T$.

The main deficiency of periodic signals which correspond to sequences $A^{(2)}_N$ ($\alpha = \alpha_2 =2$) (9) is a large peak-factor ν value, which is the ratio between peak and average signal power [2], [5]. The peak-factor for sequences $A^{(\alpha 2)}_N = A^{(2)}_N$ is

$$\nu^{(2)}_{AN}= P_{max}/P_{av}= (N-2)^2/N, \tag{17}$$

where maximum power (also known as peak-power) $P_{max} = |a^{(\alpha)}_0|^2 = (N-2)^2$ and average power $P_{av} = \sum(\alpha^{(\alpha)}_i)^2/N = R^{(2)}_{AN}(\tau=0)/N = N$.There are several ways to reduce the peak-factor ν; one way is related with the use of mismatched filters.

IV. Signal Processing Using Mismatched Filters

Consider periodic discrete sequences C_N and D_N of duration N:

$$\begin{aligned} C_N &= \{c_i\} = (c_0, c_1, c_2, \dots, c_{N-1}) = (c_0, -2, -2, -2, \dots, -2), \\ D_N &= \{d_i\} = (d_0, d_1, d_2, \dots, d_{N-1}) = (d_0, -2, -2, -2, \dots, -2). \end{aligned} \tag{18}$$

These periodic sequences can be distinguished from sequences $A^{(\alpha 2)}_N=A^{(2)}_N$ ($\alpha = \alpha_2 =2$) (9) by the value of only the first elements, i.e. the value of c_0 and d_0.

Let us assume that sequence C_N corresponds to the transmitted signal and sequence D_N corresponds to the filter which is used for processing of this signal. The cross correlation function (CCF) of these two sequences $R_{CDN}(\tau)$ corresponds to a signal on the output of the processing filter. It is possible to show that only under the condition

$$c_0 + d_0 =2(N-2) \tag{19}$$

the cross correlation function of these two sequences $R_{CDN}(\tau)$ has only one maximum when $\tau=0$ and has a zero value $R_{CDN}(\tau)=0$ when $\tau\neq0$.

In the case when $c_0 = d_0 = N-2$, sequences C_N and D_N (18) are equal and their cross correlation function $R_{CDN}(\tau)$ is the same as autocorrelation function (ACF) $R^{(\alpha)}_{AN}(\tau)$ (10)

$$R_{CDN}(\tau) = R^{(\alpha)}_{AN}(\tau), \qquad \text{if} \quad c_0=d_0 = N-2. \tag{20}$$

Decreasing the value of c_0 ($c_0 < N-2$) in the sequence C_N leads to a reduction of peak-factor of the periodic discrete signal because the sequence C_N corresponds to the signal. The filter corresponding to sequence D_N ($d_0> N-2$) and satisfying the condition (19) can be used to process this signal. In this case, the output signal of this filter, corresponding to cross correlation function $R_{CDN}(\tau)$, has the only one maximum on the interval NT and the filter is a mismatched filter.

Below we provide several samples of sequences C_N and D_N, where $c_0<N-2$ and $d_0>N-2$, and c_0 and d_0 satisfy condition (19), i.e. $c_0 + d_0 = 2(N-2)$. These samples are represented by pairs of sequences C_N and D_N which correspond to pairs of signals sequences C_N and mismatched filters sequences D_N for various N.

N=10 and $c_0 + d_0 = 2(N\text{-}2) = 16$

$c_0 = d_0 = N\text{-}2 = 8$, $C_N = (8, -2, -2, .., -2, -2)$, $D_N = (8, -2, -2, .., -2, -2)$,
$c_0 = 6$, $d_0 = 10$, $C_N = (6, -2, -2, .., -2, -2)$, $D_N = (10, -2, -2, .., -2, -2)$,
$c_0 = 4$, $d_0 = 12$, $C_N = (4, -2, -2, .., -2, -2)$, $D_N = (12, -2, -2, .., -2, -2)$,

N= 20 and $c_0 + d_0 = 2(N\text{-}2) = 36$ (21)

$c_0 = d_0 = N\text{-}2 = 18$, $C_N = (18, -2, -2, .., -2, -2)$, $D_N = (18, -2, -2, .., -2, -2)$,
$c_0 = 12$, $d_0 = 24$, $C_N = (12, -2, -2, .., -2, -2)$, $D_N = (24, -2, -2, .., -2, -2)$,
$c_0 = 6$, $d_0 = 30$, $C_N = (6, -2, -2, .., -2, -2)$, $D_N = (30, -2, -2, .., -2, -2)$,
$c_0 = 4$, $d_0 = 32$, $C_N = (4, -2, -2, .., -2, -2)$, $D_N = (32, -2, -2, .., -2, -2)$.

In all of the above samples (21), the value of N defines the length of sequences C_N and D_N.

The case $c_0 = d_0 = N\text{-}2$ ($c_0 + d_0 = 2(N\text{-}2)$) (21) corresponds to the use of a matched filter. In this case, CCF $R_{CDN}(\tau)$ and ACF $R^{(\alpha)}_{AN}(\tau)$ are the same (20). In all other cases, $c_0 < N\text{-}2$ and $d_0 > N\text{-}2$, but $c_0 + d_0 = 2(N\text{-}2)$ (21), and the processing output signals of a mismatched filter is CCF of C_N (signal) and D_N (filter). This CCF has only one maximum on interval NT without any side lobes. A single maximum without any side lobes means that these output signals corresponds to perfect ACF, however the peak-factor for signal C_N is lower because c_0 is lower than N-2.

In the case of additive white Gaussian noise (AWGN) channels, the use of mismatched filters leads to losses of signal to noise ratio (SNR) at the output of mismatched filters compared to the SNR of matched filters, i.e. filters when $c_0 = d_0 = N\text{-}2$. The decrease of SNR is a result of the decrease of the maximum output signal values of mismatched filters compared with the maximum output signal values of matched filters. The decrease of SNR is also a result of the increase of noise power on the output of mismatched filters compared with noise power on the output of matched filters. The SNR ratio on the outputs of mismatched filters can be calculated using methods in [2] and [5].

By analyzing these cases, we observe that signal processing using mismatched filters is related with the use of sidelobe suppression filters [2], [5] and [10] for binary signals. When $c_0 = 2$ for any value of N, all input signals which correspond to the sequences $C_N = (c_0, -2, -2, -2, \ldots, -2) = (2, -2, -2, -2, \ldots, -2)$ (18) are binary signals (Fig. 1c). The ACF of these signals have side lobes on interval NT and by processing these signals with filters corresponding to sequences $D_N = (d_0, d_1, d_2, \ldots, d_{N-1}) = (d_0, -2, -2, -2, \ldots, -2)$, where $d_0 = 2(N\text{-}2) - c_0 = 2(N\text{-}3)$ (19), it is possible to obtain filter output signals which correspond to signals with perfect ACF; i.e. these output signals have only one maximum without any side lobes. Filters with these properties are called side lobe suppression filters (SLSF) [2], [5], and [10]. Below we provide several examples of sequences $C_N = (2, -2, -2, -2, \ldots, -2)$ which correspond to binary signals and sequences $D_N = (d_0, -2, -2, -2, \ldots, -2)$ which correspond to processing SLSFs where $d_0 = 2(N\text{-}2) – c_0 = 2(N\text{-}3)$.

$$\begin{array}{lll}
\mathbf{N=5,} & C_N = (2, -2, -2, -2, -2), & D_N = (4, -2, -2, -2, -2), \\
\mathbf{N=7,} & C_N = (2, -2, -2, -2, -2, -2, -2), & D_N = (8, -2, -2, -2, -2, -2, -2), \\
\mathbf{N=10,} & C_N = (2, -2, -2, -2, \ldots, -2, -2), & D_N = (14, -2, -2, -2, \ldots, -2, -2), \\
\mathbf{N=20,} & C_N = (2, -2, -2, -2, \ldots, -2, -2), & D_N = (34, -2, -2, -2, \ldots, -2, -2), \\
\mathbf{N=80,} & C_N = (2, -2, -2, -2, \ldots, -2, -2), & D_N = (154, -2, -2, -2, \ldots, -2, -2), \\
\mathbf{N=500,} & C_N = (2, -2, -2, -2, \ldots, -2, -2), & D_N = (994, -2, -2, -2, \ldots, -2, -2).
\end{array} \quad (22)$$

In all of the above samples (22), the length of sequences C_N and D_N equals N. When $c_0 = 2$, these signals are binary signals (Fig. 1c) and for periodic binary signals the peak-factor ν (15) equals 1.

It is clear that the use of SLSFs also leads to some losses related with the increase of SNR at the filter outputs [2], [5].

V. Conclusion

This paper presents a new method for designing periodic discrete non-binary signals with perfect ACF. When applied to radar systems, these signals provide high resolution distance (time delay) measurements together with high resolution Doppler frequency shift (speed) measurements. It is shown that when using mismatched filters for processing periodic non-binary signals, it is possible to reduce the peak-factor ν of these signals.

Appendix

Cyclic shifted periodic sequences $A^{(\alpha)}_N = \{a^{(\alpha)}_i\} = ((N-\alpha), -\alpha, -\alpha, -\alpha, \ldots\ldots, -\alpha)$

Table 1

τ	**$\{a^{(\alpha)}_{i-\tau}\}$**	**$(a^{(\alpha)}_{i-\tau\ (\mathrm{mod}\ N)})$**
0	$\{a^{(\alpha)}_i\}$	$(N-\alpha), -\alpha, -\alpha, \ldots\ldots, -\alpha, -\alpha$
1	$\{a^{(\alpha)}_{i-1}\}$	$-\alpha, (N-\alpha), -\alpha, \ldots\ldots, -\alpha, -\alpha$
2	$\{a^{(\alpha)}_{i-2}\}$	$-\alpha, -\alpha, (N-\alpha), \ldots\ldots, -\alpha, -\alpha$
… **…**	…… ……	………… …………
N-1	$\{a^{(\alpha)}_{i-(N-1)}\}$	$-\alpha, -\alpha, -\alpha, \ldots\ldots, -\alpha, (N-\alpha)$

References

1. Pingzhi Fan and Michael Darnell. “Sequence Design for Communication Applications”. Research Study Press Ltd., England. 1996.
2. Valery P. Ipatov. “Spread Spectrum and CDMA: Principles and Applications”. John Wiley & Sons Ltd., England, 2005.
3. K. J. Horadam. “Hadamard Matrices and Their Applications”. Princeton University Press, 2007.
4. P. M. Woodward. “Probability and Information Theory with Applications to Radar”. London, Pergamon Press, 1953.
5. Nadav Levanon, Eli Mozeson. “Radar Signals”. John Wiley & Sons, Inc. USA, 2004.
6. Mark A. Richard. “Fundamentals of Radar Signal Processing”. McGraw-Hill Education, Second Edition, USA, 2014.
7. Nadav Levanon . “The Periodic Ambiguity Function-Its Validity and Value.” 2010 IEEE Radar Conference, 10-14 May, Washington DC, pp. 204-208, 2010.
8. Dylan Eustice, Charles Baylis and Robert J. Marks II. “Woodward Ambiguity Function: From Foundations to Applications”, 2015 IEEE Texas Symposium on Wireless and Microwave Circuits and Systems (WMCS), April 23-24, Waco, Texas (pp. 1-17).
9. H. Rohling and W. Plagge. “Mismatched-filter design for periodical binary phased signals”. IEEE Trans. On Aerospace and Electronic Systems, AES-25(6), pp. 890-897, November 1989.
10. Ackroyd, M. H. and Ghani, F. “Optimal mismatched filter for sidelobe suppression”. IEEE Trans. On Aerospace and Electronic Systems, 9, pp. 214-218, 1973.
11. R. H. Barker. “Group synchronizing of binary digital systems.” Communication theory, London, 1953, pp. 273-287.
12. M.V. Zaleshin and V. E. Gantmakher. “About a family of almost four-phased sequences with perfect periodic autocorrelation function”. ARPN Journal of Engineering and Applied Sciences, vol. 11, No. 10, May 2016.

AUTHOR DETAILS

Alexander V. Titov (M’14) is a retired scientist. He graduated in 1962 from the Electrotechnical University LETI, St. Petersburg (formerly Leningrad), Russia (www.eltech.ru) with a Master’s Degree in Radiotechnology and received his PhD in 1967. He was an Associate Professor at Electrotechnical University LETI (1969-1990). His primary field of interest was digital signal processing. He has several inventions and publications in scientific journals. He participated in the IREX (International Research and Exchange) Program working at the Valley Forge Research Center (with Prof. B. Steinberg) of the University of Pennsylvania, Philadelphia, PA from February-December 1974. He is a life member of the International Loran Association (ILA), a member of IEEE and is now retired and living in the USA.

Gregory J. Kazmierczak (M’86) is currently an independent cybersecurity consultant. He was recently the Chief Technology Officer for Wave Systems Corp., Princeton, NJ 08536 USA. He graduated from the University of Pennsylvania with a BSE in Electrical Engineering in 1984 and a MSE in Electrical Engineering in 1986 where his primary field of interest was analog integrated circuit design. He worked at IBM and Siemens Corporate Research focused on Si and GaAs digital analog integrated circuits. At Wave Systems, his primary field of interest was leveragingroots of trust and hardware-based security devices to protect user and enterprise electronic assets. Mr. Kazmierczak represented Wave Systems at standards organizations such as Trusted Computing Group, IETF, IEEE and OASIS. He has been granted four patents and is a member of IEEE.