ISSN: 2348 - 2117

International Journal of Engineering Technology and Computer Research (IJETCR)
Available Online at www.ijetcr.org
Volume 4; Issue 3; May-June-2016; Page No. 12-18

BUCKLING OF SIGMOID TIMOSHENKO BEAM ON 2-PARAMETER VARIABLE ELASTIC FOUNDATION

S.N PADHI[1], R.C.MOHANTY[2], T.ROUT[*]

[1]Department of Mechanical Engineering, Centurion University of Technology&Management,Odisha.

snpadhi333@gmail.com

[2]Department of Mechanical Engineering, Centurion Institute of Technology,Bhubaneswar, Odisha.

rcmohanty@cutm.ac.in

[*]Department of Mechanical Engineering, Parala Maharaja Engineering College, Berhampur, Odisha.

troutwala@gmail.com

Abstract

A sigmoid Timoshenko beam resting on two parameter variable elastic foundation is investigated for its buckling behavior. Finite element method is used for simply supported end condition of the beam. The static part of the governing differential equation of the beam is solved to determine the shape functions to obtain accurate results. The effect of geometry, power index, foundation stiffness and foundation parameter on critical buckling load of the beam is investigated for various foundations.

Key words: *sigmoid, foundation parameter, foundation modulus, power index sinusoidal foundation.*

1. INTRODUCTION:

Functionally graded material (FGM) is an advanced type of composite material which consists of two or more dissimilar materials and the composition of this material varies continuously with respect to spatial coordinates thereby varying its properties accordingly. FGMs can be used in many engineering sectors such as the aerospace, aircraft, automobile, defense industries, electronic and the biomedical sectors. Many machine and structural components in aforesaid sections can be modeled as beams subjected to environmental forces in both static as well as dynamic state.When the load on the beam reaches a critical value, the beam might buckle and becomes unstable. A literature survey in this regard is carried out and an overview of the survey is given below.

Zhang& Zhou.[11] have studied the mechanical and thermal post-buckling analysis of FGM rectangular plates resting on nonlinear elastic foundations using the concept of physical neutral surface and high-order shear deformation theory, and investigated the post-buckling behavior of FGM rectangular plates with two opposite simply supported edges and other two opposite clamped edges using multi-term Ritz method. Sofiyev, A. H.[7] studied the buckling analysis of functionally graded material (FGM) circular truncated conical and cylindrical shells subjected to combined axial extension loads and hydrostatic pressure and resting on a Pasternak type elastic foundation and found analytically the critical combined loads of FGM truncated conical shells with or without elastic foundations. Duc& Thang[4] presented an analytical approach to investigate the nonlinear static buckling and post-buckling for imperfect eccentrically stiffened functionally graded thin circular cylindrical shells surrounded on elastic foundation with ceramic-metal-ceramic layers (S-FGM) and subjected to axial compression. Ghiasian,S.E. et al[5]have studied the static and dynamic buckling of an FGM beam subjected to uniform temperature rise loading and uniform compression and obtained Nonlinear governing equations based on the static version of virtual displacements and solved via the multi-term Galerkin method. He also estimated the dynamic buckling load levels based on the well-known Hoff-Simitses criterion and found that for sufficiently stiff softening elastic foundation, post-buckling equilibrium path becomes unstable. Furthermore, when the thermal post-buckling equilibrium path is stable, no dynamic buckling occurs according to this criterion. Sofiyev, A. H.[8] focused on the thermal buckling analysis of FGM shells resting on the two-parameter elastic foundation and graded the material properties of the constituents inthe thickness direction according to the power-law distribution with the surrounding elastic medium is modeled as an elastic foundation of the Pasternak-type.

Bagherizadeh,E. et. al.[1] have investigated the mechanical buckling of functionally graded material cylindrical shell that is embedded in an outer elastic medium and subjected to combined axial and radial compressive loads assuming the material properties to vary smoothly through the shell thickness according to a power law distribution of the volume fraction of constituent materials and modelled the elastic foundation by two parameters Pasternak model obtained by adding a shear layer to the Winkler model considering the simply-supported boundary condition. Shariyat& Asemi[6]have studied the shear buckling analysis of the orthotropic heterogeneous FGM plates for the first time and considered the influence of the Winkler-type elastic foundation. Yang,J. et al[10] have investigated the effect of randomness in properties on the elastic buckling of FGM rectangular plates which are resting on an elastic foundation and subjected to uniform in-plane edge compressions including the interaction between the plate and foundation in the formulation with a two-parameter Pasternak model. Swaminathan, K. et al[9] have made a comprehensive review of the various methods employed to study the static, dynamic and stability behavior of Functionally Graded Material (FGM) plates by both analytical and numerical methods with an emphasis to present stress, vibration and buckling characteristics of FGM plates predicted using different theories proposed by several researchers without considering the detailed mathematical implication of various methodologies. Duc&Quan[3] have presented the nonlinear response of eccentrically stiffened FGM cylindrical panels on elastic foundation subjected to mechanical loads and determined the explicit relations of load-deflection curves for simply supported eccentrically stiffened FGM panels by applying Bubnov-Galerkin method, the Lekhnitsky smeared stiffeners technique with Pasternak type elastic foundation and stress function.

Though the literatures on static and dynamic stability of isotropic beams are plenty, the literature on sigmoid Timoshenko beams on 2-parameter variable elastic foundations have rarely been reported to the best of the authors' knowledge. In the present article, FGO beams hinged at both the ends and resting on 2-parameter variable elastic foundation is considered for buckling analysis.

2. Formulation:

A functionally graded sigmoid beam with steel and aluminum as its constituent phases is considered for analysis as shown in Fig.1(a). The beam, hinged at both the ends is subjected to a dynamic axial load $P(t) = P_s + P_d \cos \Omega t$. Where, t is time, P_s is the static component, P_d is the amplitude of the dynamic component and Ω is the frequency of the applied dynamic load component of $P(t)$. The mid-longitudinal(x-y) plane is chosen as the reference plane for expressing the displacements as shown in fig. 1(b). The thickness coordinate is measured as z from the reference plane. The axial displacement, the transverse displacement, and the rotation of the cross section are u , w and φ respectively. Fig. 1(c) shows a two nodded beam finite element having three degrees of freedom per node.

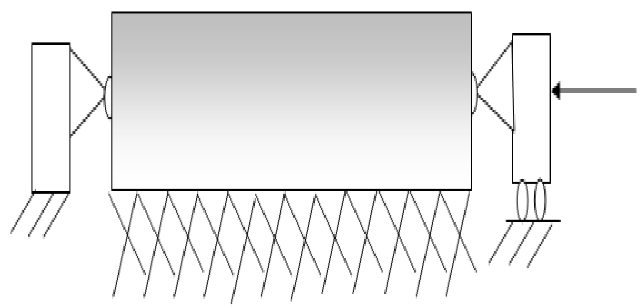

Figure 1 (a): Functionally graded sandwich beam subjected to dynamic axial load.

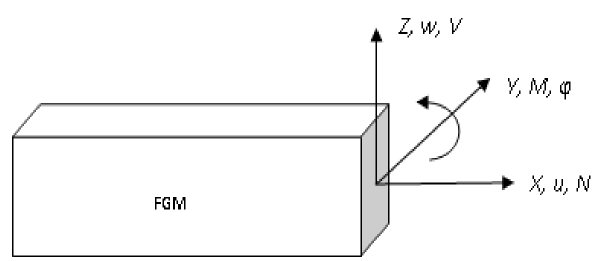

Figure 1 (b): The coordinate system with generalized forces and displacements for the FGSW beam element.

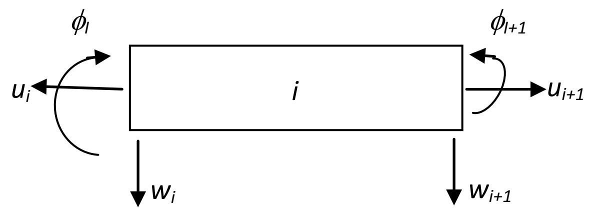

Figure 1 (c): Beam element showing generalized degrees of freedom for i^{th} element.

The generalized displacement vector of the element can be given as

$$\{\hat{u}\} = [u_i \;\; w_i \;\; \varphi_i \;\; u_{i+1} \;\; w_{i+1} \;\; \varphi_{i+1}] \qquad (2.1)$$

The equation of motion for the element subjected to axial force $P(t)$ can be expressed in terms of nodal degrees of freedom as

$$[m]\{\ddot{\hat{u}}\} + \left[[k_{ef}] - P(t)[k_g]\right]\{\hat{u}\} = 0 \qquad (2.2)$$

The axial load $P(t)$ is taken as $P(t) = \alpha P^{\bullet} + \beta_d P^{\bullet} \cos\Omega t$,

so that, $P_s = \alpha P^{\bullet}$ and $P_d = \beta_d P^{\bullet}$. $P^{\bullet}$ is the critical buckling load of an isotropic beam with similar geometrical dimensions and end conditions and α, β_d are called static and dynamic load factors respectively. Considering the application of static and dynamic component of load in the same manner we have eq.(2.2) in the following form

$$[m]\{\ddot{\hat{u}}\} + \left[[k_{ef}] - P^{\bullet}(\alpha + \beta_d \cos\Omega t)[k_g]\right]\{\hat{u}\} = 0 \qquad (2.3)$$

$$[k_{ef}] = [k_e] + [k_f]$$

where, $[k_e]$, $[k_f]$, $[m]$ and $[k_g]$ are element elastic stiffness matrix, foundation stiffness matrix, mass matrix and geometric stiffness matrix respectively. Assembling the element matrices as used in eq. (2.3), the equation in global matrix form which is the equation of motion for the straight beam, can be expressed as

$$[M]\{\ddot{\hat{U}}\} + \left[[K_{ef}] - P^{\bullet}(\alpha + \beta_d \cos\Omega t)[K_g]\right]\{\hat{U}\} = 0 \qquad (2.4)$$

$$[K_{ef}] = [K_e] + [K_f]$$

$[M]$, $[K_e]$, $[K_f]$, $[K_g]$ are global mass, elastic stiffness, foundation stiffness and geometric stiffness matrices respectively and $[\hat{U}]$ is global displacement vector. Equation (2.4) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The periodic solutions for the boundary between the dynamic stability and instability zones can be obtained fromFloquet Theory as follows. A solution with twice the time period which is of practical importance is represented by

$$\hat{U}(t) = c_1 \sin\frac{\Omega t}{2} + d_1 \cos\frac{\Omega t}{2}, \qquad (2.5)$$

considering first order expansion.

Substituting eq. (2.5) into eq. (2.4) and comparing the coefficients of $\sin\frac{\Omega t}{2}$ and $\cos\frac{\Omega t}{2}$ terms the condition for existence of these boundary solutions with twice the time period is given by

$$\left([K_{ef}] - (\alpha \pm \beta_d/2)P^{\bullet}[K_g] - \frac{\Omega^2}{4}[M]\right)\{\hat{U}\} = 0 \qquad (2.6)$$

Equation (2.6) represents an eigenvalue problem for known values of α, β_d, and $P^{\bullet}$. This equation gives two sets of eigenvalues (Ω) binding the regions of instability due to the presence of plus and minus sign. The instability boundary can be determined from the solution of the equation

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\bullet}[K_g] - \frac{\Omega^2}{4}[M]\right| = 0 \qquad (2.7)$$

2.1 Critical buckling load:

When α =1, β_d =0, and $\omega = 0$, eq. (2.7) is reduced to a problem of static stability.

$$\left|[K_{ef}] - P^{\bullet}[K_g]\right| = 0 \qquad (2.8)$$

The solution of eq. (2.8) gives the value of critical buckling load.

3 Element matrices:

The element matrices for the SFG beam element are derived following the procedure as proposed by Chakraborty,A. et al[2].

3.1 Shape functions:

The displacement fields according to first order shear deformation beam theory is expressed as

$$U(x,y,z,t) = u(x,t) - z\phi(x,t), \;\; W(x,y,z,t) = w(x,t), \qquad (3.1)$$

The cross-sections are assumed to remain plane after the deformation.

The longitudinal and shear strains are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial \varphi}{\partial x}, \;\; \gamma_{xz} = -\varphi + \frac{\partial w}{\partial x} \qquad (3.2)$$

where $\frac{\partial w}{\partial x}$ is the slope of the deformed longitudinal axis`

The stress-strain relation in matrix form can be given by

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} E(z) & 0 \\ 0 & kG(z) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} \qquad (3.3)$$

Where σ_{xx} is the normal stress in longitudinal direction and τ_{xz} is shear stress in $x - z$ plane, $E(z)$ is Young's modulus and $G(z)$ is shear modulus and k is shear correction factor.

The material properties of the FGM that varies along the thickness of the beam are assumed to follow sigmoid distribution given by

$$R(z) = R_t g_1(z) + R_b(1 - g_1(z)) \qquad 0 \le z \le h/2,$$

$$R(z) = R_t g_2(z) + R_b(1 - g_2(z)) \qquad -h/2 \le z \le 0 \; (3.4)$$

$$g_1(z) = 1 - \frac{1}{2}\left(1 - \frac{2z}{h}\right)^n \qquad g_2(z) = \frac{1}{2}\left(1 + \frac{2z}{h}\right)^n \qquad (3.5)$$

where, $R(z)$ denotes a material property such as, E, G, ρ etc., R_t and R_b denote the values of the properties at topmost and bottommost layer of the beam respectively, and n is an index.

The kinetic energy T and elastic strain energy S of an element are given respectively as

$$T = \frac{1}{2}\int_0^l \int_A \rho(z)\left[\left(\frac{\partial U}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right] dA dx \qquad (3.6)$$

$$S = \frac{1}{2}\int_0^l \int_A E(z)\left[\left(\frac{\partial u}{\partial x}\right)^2 + z^2\left(\frac{\partial \varphi}{\partial x}\right)^2 - 2z\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \varphi}{\partial x}\right)\right] dA dx + \frac{1}{2}\int_0^l \int_A G(z)\left[\varphi^2 + \left(\frac{\partial w}{\partial x}\right)^2 - 2\varphi\frac{\partial w}{\partial x}\right] dA dx \qquad (3.7)$$

The governing differential equations can be derived by applying Hamilton's principle as presented below.

$$\frac{\partial(T-S)}{\partial u} = 0, \quad \frac{\partial(T-S)}{\partial w} = 0, \text{ and } \frac{\partial(T-S)}{\partial \varphi} = 0 \qquad (3.8)$$

The shape functions for the displacement field for finite element formulation are obtained by solving the static part of the eq. (3.8) with the following consideration.

$$u = a_1 + a_2 x + a_3 x^2,$$

$$w = a_4 + a_5 x + a_6 x^2 + a_7 x^3,$$

$$\varphi = a_8 + a_9 x + a_{10} x^2. \qquad (3.9)$$

The above displacement fields are substituted in the static part of Eq.(3.8) in order to find out the constants of polynomials. Subsequently, the displacement fields are expressed in terms of the nodal degree of freedoms as follows.

$$\{\bar{u}\} = [u \;\; w \;\; \varphi]^T = [\aleph(x)]\{\hat{u}\} \qquad (3.10)$$

$\aleph(x)$, a 3x6 matrix is the required shape function. Better convergence can be achieved as the shape functions are obtained from the exact solution of static part of the governing differential equation. Now the shape function can also be expressed as

$$\aleph(x) = [\aleph_u(x) \;\; \aleph_w(x) \;\; \aleph_\varphi(x)]^T. \qquad (3.11)$$

where, $\aleph_u(x)$, $\aleph_w(x)$, $\aleph_\varphi(x)$ are the shape functions for the axial, transverse and rotational degree of freedom respectively.

3.2 Element elastic stiffness matrix:

The general force boundary conditions for the element can be given as

$$N_x = \int_A \sigma_{xx} dA = A_{11}\frac{\partial u}{\partial x} - B_{11}\frac{\partial \varphi}{\partial x}$$

$$V_x = \int_A \tau_{xz} dA = A_{55}\left(\frac{\partial w}{\partial x} - \varphi\right)$$

$$M_x = -\int_A z\sigma_{xx} dA = -B_{11}\frac{\partial u}{\partial x} + D_{11}\frac{\partial \varphi}{\partial x} \qquad (3.12)$$

where, N_x, V_x, M_x are axial force, shear force and bending moment respectively acting at the boundary nodes. Similarly substituting eq. (3.10) into eq. (3.12) we get

$$[k_e]\{\hat{u}\} = \{F\} \qquad (3.13)$$

Where

$$\{F\} = [-N_x(0) \;\; -V_x(0) \;\; -M_x(0) \;\; N_x(l) \;\; V_x(l) \;\; M_x(l)]^T$$

is the nodal load vector and

$[k_e]$ is the required element elastic stiffness matrix.

3.3 Element elastic foundation matrix:

The work done by the foundation is given by the expression

$$W_f = \frac{1}{2}\int_0^l k(x) w^2 dx + \frac{k_p}{2}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx$$

$$= \frac{1}{2}\int_0^l \{\hat{u}\}^T k(x)[\aleph_w]^T[\aleph_w]\{\hat{u}\} dx + \frac{1}{2}\int_0^l \{\hat{u}\}^T k_p [\aleph_w]'^T [\aleph_w]' \{\hat{u}\} dx$$

$$= \frac{1}{2}\{\hat{u}\}^T [k_f]\{\hat{u}\} \qquad (3.14)$$

Where, k is the foundation stiffness parameter per unit width of the beam and

$[k_f] = \int_0^l k(x)[\aleph_w]^T[\aleph_w]dx + \int_0^l k_p[\aleph_w]'^T[\aleph_w]' dx$ is element foundation stiffness matrix. The first part is the foundation stiffness matrix resulted due to resistance of foundation against transverse deflection and the second part is the foundation stiffness resulted due to interaction of the shear layer of the foundation with the mating bottom surface of beam. Here k_p is assumed to be constant.

The foundation stiffness that varies along the length of beam considered in present study is as follows.

$k = k_o(1-\psi x)$ for linear variation of foundation stiffness

$k = k_o(1-\xi x^2)$ for parabolic variation of foundation stiffness.

$k = k_o(1-\mu \sin x)$ for sinusoidal variation of foundation stiffness.

3.4 Element geometric stiffness matrix:

When an axial load P is applied on the beam element, the work done by the load can be expressed as

$$W_p = \frac{1}{2}\int_0^l P(t)\left(\frac{\partial w}{\partial x}\right)^2 dx \tag{3.15}$$

Substituting the value of w from eq. (3.10) into eq. (3.15) the work done can be expressed as

$$W_p = \frac{P(t)}{2}\int_0^l \{\hat{u}\}^T[\aleph'_w]^T[\aleph'_w]\{\hat{u}\}dx$$

$$= \frac{P(t)}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \tag{3.16}$$

where, $[k_g] = \int_0^l [\aleph'_w]^T[\aleph'_w]dx$ is called the element geometric stiffness matrix.

4 Results and discussion:

The simulation is carried out for a sigmoid functionally graded (SFG) beam for simply supported end conditions and resting on variable elastic foundations. The constituent phases chosen are steel and aluminum. The critical buckling load is non-dimensionalised by that of Euler beam of length 0.5m, width 0.1m and thickness 0.125m of same end conditions.

Analysis of critical buckling load:

An SFG beam with steel-rich bottom is considered for analysis of static stability. The length of the beam is 0.5m, width is 0.1 m with various thicknesses. The material properties are as follows.

Steel: E =2.1x10^{11} Pa, G =0.8x10^{11} Pa ρ =7.85x10^{3}kg/m^{3}. The shear correction factor k = 0.8667.

Aluminium: E =0.7x10^{11} Pa, G =0.2697x10^{11} Pa, ρ =2.707x10^{3}kg/m^{3}.

The following non-dimensional numbers are used for analysis purpose.

The non-dimensional natural frequencies, $\eta_1 = \omega_1\left(\frac{\rho A L^4}{EI}\right)^{1/2}$,

the non-dimensional foundation modulus, $K_1 = \frac{k_v L^4}{EI}$ andthe foundation shear modulus, $K_2 = \frac{k_p L^2}{\pi^2 EI}$

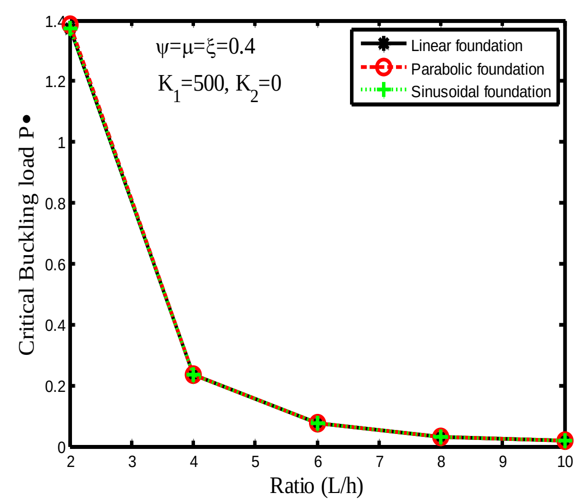

Figure 2: Effect of geometry on critical buckling load of beam resting on variable elastic foundations.

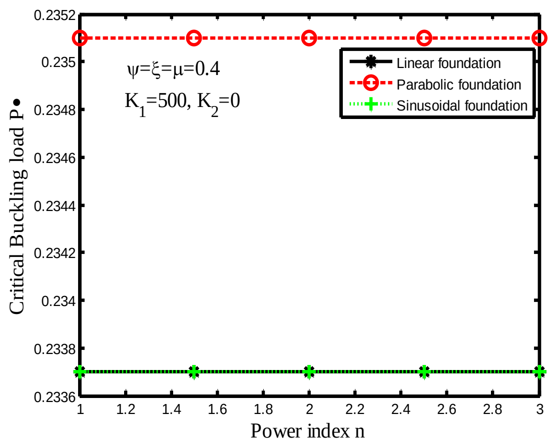

Figure 3: Effect of power index on critical buckling load of beamresting on variable elastic foundations.

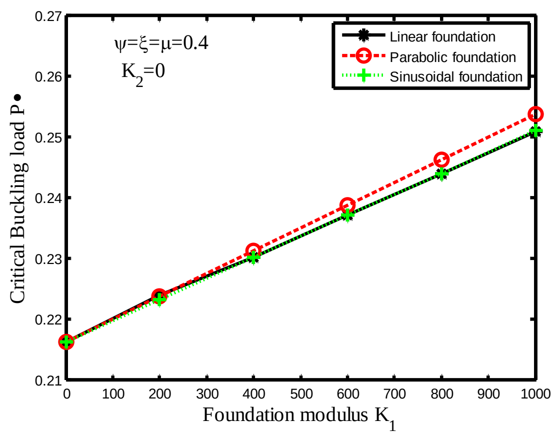

Figure 4: Effect of resistance of various foundations against deflection of sigmoid beam on its buckling.

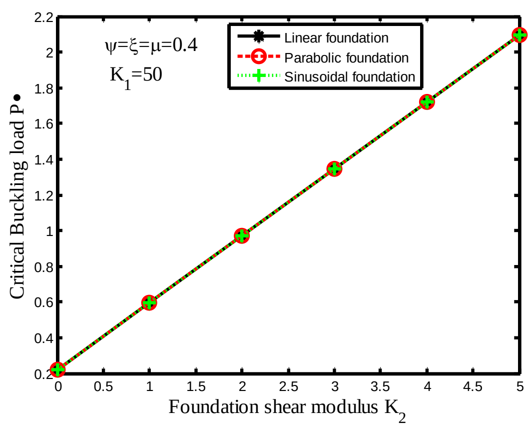

Figure 5: Effect of interaction of shear layer of variable foundations on buckling of sigmoid beam.

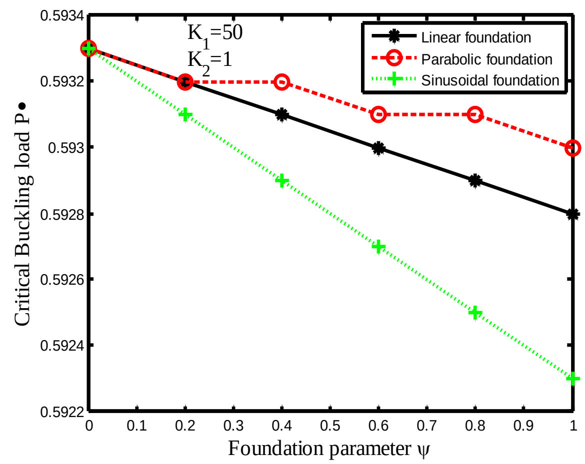

Figure 6: Comparison among the variable elastic foundations regarding their effect on the buckling of sigmoid beam.

Fig. 2 reveals the effect of slenderness parameter(L/h) on critical buckling load of beam with steel-rich bottom for property distribution according to power index n=1. It is observed that the buckling load decreases sharply with increase of slenderness parameter for all the foundations considered. This is due to the fact that with increase in slenderness parameter the beam approaches the Euler beam which makes it more prone to buckling. Moreover, the buckling load of beam resting on parabolic foundation is the slightly higher as compared to other foundations. The effect of material properties on critical buckling load is presented in fig. 3. It is observed that the variation of power index has no bearing on the buckling load. This is obvious as the sigmoid beam remains symmetric irrespective of the power indices. The effect of foundation on critical buckling load is investigated and shown in fig. 4 and fig. 5. Fig. 4 presents the contribution of foundation by resisting the deflection of beam and the effect of interaction of shear layer of foundation is shown in fig. 5.The static stability increases with increase of foundation modulus and foundation shear modulus as well as shown in fig. 4 and fig. 5 respectively.

It is observed from fig. 4 and fig. 5 that the contribution of shear layer of beam in improving the stability behavior of beam is more significant. Fig. 6 presents the comparison among various foundations as regards their role on stability behavior of beam. The parabolic foundation renders highest stability to the beam while the sinusoidal foundation causing the lowest buckling loads of beam.

5. Conclusion:

Finite element method is used to investigate the buckling of SFG beam resting on variable elastic foundation such as linear parabolic and sinusoidal. The following conclusions may be drawn from the above analysis:

- Foundation improves the stability behavior of the beam.
- Effect of interaction of foundation on buckling of beam is more prominent.
- Sigmoid distribution for designing FGM beams ensures no change in stability behavior under variation of power indices.

Reference:

1. Bagherizadeh,E.,Kiani,Y. andEslami, M. R. “Mechanical buckling of functionally graded material cylindrical shells surrounded by Pasternak elastic foundation,” *Compos. Struct.*, vol. 93, no. 11, pp. 3063–3071, 2011.
2. Chakraborty,A,Gopalakrishnan,S. and Reddy, J.N. “A new beam finite element for the analysis of functionally graded materials”, International Journal of Mechanical Science 45 (2003) 519-539.
3. Duc,N. D. and Quan,T. Q. “Nonlinear response of imperfect eccentrically stiffened FGM cylindrical panels on elastic foundation subjected to mechanical loads,” *Eur. J. Mech. A/Solids*, vol. 46, pp. 60–71, 2014.
4. Duc,N. D. and Thang,P. T. “Nonlinear response of imperfect eccentrically stiffened ceramic-metal-ceramic FGM thin circular cylindrical shells surrounded on elastic foundations and subjected to axial compression,” *Compos. Struct.*, vol. 110, no. 1, pp. 200–206, 2014.
5. Ghiasian,S.E.,Kiani,Y.,andEslami,M.R. “Dynamic buckling of suddenly heated or compressed FGM beams resting on nonlinear elastic foundation,” *Compos. Struct.*, vol. 106, pp. 225–234, 2013.
6. Shariyat,M. and Asemi,K.“Three-dimensional non-linear elasticity-based 3D cubic B-spline finite element shear buckling analysis of rectangular orthotropic FGM plates surrounded by elastic foundations,” *Compos. Part B Eng.*, vol. 56, pp. 934–947, 2014.
7. Sofiyev, A. H. “Buckling analysis of FGM circular shells under combined loads and resting on the Pasternak type elastic foundation,” *Mech. Res. Commun.*, vol. 37, no. 6, pp. 539–544, 2010.
8. Sofiyev, A. H. “Thermal buckling of FGM shells resting on a two-parameter elastic foundation,” *Thin-Walled Struct.*, vol. 49, no. 10, pp. 1304–1311, 2011.
9. Swaminathan,K., Naveenkumar,D.T.,Zenkour,A. M. and Carrera,E. “Stress, vibration and buckling analyses of FGM plates-A state-of-the-art review,” *Composite Structures*, vol. 120. pp. 10–31, 2015.
10. Yang, J.,Liew,K. M. and Kitipornchai,S. “Second-order statistics of the elastic buckling of functionally graded rectangular plates,” *Compos. Sci. Technol.*, vol. 65, no. 7–8, pp. 1165–1175, 2005.
11. Zhang,D. G. and Zhou, H. M. “Mechanical and thermal post-buckling analysis of FGM rectangular plates with various supported boundaries resting on nonlinear elastic foundations,” *Thin-Walled Struct.*, vol. 89, pp. 142–151, 2015.