

Survey of Image Denoising Methods using Double-Density Complex DWT

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ABSTRACT

Image denoising means removal of noise from an image and collect the remaining information from that image. A wavelet Transformation method is very widely used now days for noise removal. It provides multi resolution representation using a set of analyzing functions that are dilations and translations of a few functions. but the extensions of DWT are now widely used . In this paper basic introduction about the wavelet transformation and its extensions are provided. The wavelet based denoising method uses DWT (Discrete Wavelet Transform) suffers a lack of directionality and shift variance. To overcome this, CWT (Complex Wavelet Transform) can be used in two modes namely, Dual-Tree Complex DWT and Double-Density Complex DWT. In this paper, these two modes are compared.

Key Words: DWT, Dual-Tree Complex DWT, Double Density DWT

INTRODUCTION

Generally in images, noise removal is a very important. For removal of noise the main focus is on noise reduction and how much the particular image is preserved. Anything that deteriorates the quality an image is called noise. An image can be affected by noise in its capture, acquisition and processing. Generally, noise removal is an important part of image processing systems. A good image denoising model is one which will remove noise while preserving edges and contours. The wavelet transform is a simple and elegant tool that can be used for many digital signal and image processing applications. The wavelet transform provides a multiresolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). It overcomes some of the limitations of the Fourier transform with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts [30].

The wavelets Transform are of different types. The critically-sampled form of the wavelet transforms provides the most compact representation; however, it has various limitations. It does not have shift-invariance property, and in multiple dimensions it has a problem of distinguishing orientations, which is necessary in image processing. that is why, it turns out that improvements

can be obtained by using an expansive wavelet transform [32].

WAVELET TRANSFORM:

The Fourier Transform can only retrieve the global frequency content of a signal but the time information is lost. The solution of this problem is given by short time Fourier transform which calculates the Fourier transform of a windowed part of the signal and shifts the window over the signal. The STFT determines the time-frequency content of a signal with a constant frequency and time resolution because of fixed window length. But the problem still exists. For low frequencies often a good frequency resolution is required over a good time resolution. For high frequencies, the time resolution is more important. A multi-resolution analysis becomes possible by using wavelet analysis. The various types of wavelet transform are given in this paper.

A. Continuous Wavelet Transform

It is almost similar to the Fourier transform, by the convolution between the signal and analysis function .But the analysis functions are replaced by a wavelet function. The continuous wavelet transform obtains the time-frequency content information from the noisy image with an improved resolution compared to the Short Time Fourier transform.

B. Discrete Wavelet Transform

The filter banks are used for the wavelet analysis. The DWT decomposes the signal into wavelet coefficients and after that reconstruction is done. The wavelet coefficients represent the signal in various frequency bands. The coefficients can be processed different ways, giving the DWT attractive properties over linear filtering. There are two factors which represent the real line they are dilation factor and translation factor. For a particular dilation a and translation b , the wavelet coefficient $W_f(a, b)$ for a signal f can be calculated as,

$$W_f(a, b) = \{f, \psi_{a,b}\} = \int f(x) \psi_{a,b}(x) dx \quad \dots \dots (1)$$

Where, $f(x)$ is the original signal and $\psi_{a,b}(x)$ is the wavelet function. Wavelet coefficients represent the information present in the signal at the corresponding dilation and translation. The original signal can be reconstructed by applying the inverse transform:

$$f(x) = \frac{1}{C_W} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a, b) \psi_{a,b}(x) db \frac{da}{a^2} \quad \dots \dots (2)$$

Where, C_W is the normalization factor of the mother wavelet and $\psi_{a,b}(x)$ is the wavelet function [24]. The term discrete wavelet transform (DWT) uses several different methods. It must be noted that the signal itself is continuous; discrete refers to discrete sets of dilation and translation factors and discrete sampling of the signal. At a given scale J , a finite number of translations are used in applying multi resolution analysis to obtain a finite number of scaling and wavelet coefficients [6]. The signal can be represented in terms of these coefficients as,

$$f(x) = \sum_k c_{jk}(x) \phi_{jk}(x) + \sum_{j=1}^J \sum_k d_{jk} \psi_{jk}(x) \quad \dots \dots (3)$$

Where c_{jk} the scaling coefficients, d_{jk} are the wavelet coefficients, $\phi_{jk}(x)$ is the scaling function and $\psi_{a,b}(x)$ is the wavelet function. The first term in Equation (3) gives the low-resolution approximation of the signal while the second term gives the detailed information at resolutions from the original down to the current resolution J [24].

C. Complex Wavelet transform

In image and signal processing wavelet decompositions, based on separable, filters are widely used now days, mainly for data compression. Kingsbury introduced a very specific computational structure, the dual-tree complex wavelet transform [17], which shows the shift invariant properties. Other constructions can be found such as in [26]-[28]. The lack of shift invariance is the main problem in such decompositions. A manifestation of this is that coefficient power may dramatically redistribute itself

throughout sub bands when the input signal is shifted in time or in space. For perfect reconstruction the problem with Complex wavelets is the designing of complex filters. B, Kingsbury provided the solution by proposing a dual-tree implementation of the CWT (DT CWT) [19], which uses two trees:

Real filters to generate the real parts of wavelet

Imaginary filters to generate the Imaginary parts of wavelet.

The DWT suffers from the following two problems.

i] Lack of shift invariance - this results from the down sampling operation at each level. When the input signal is shifted slightly, the amplitude of the wavelet coefficients varies so much.

ii] Lack of directional selectivity - as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions.

The use of wavelets in other areas of image processing is affected by these problems. The first problem can be avoided if the filter outputs from each level are not down sampled but this increases the computational costs significantly and the resulting undecimated wavelet transform still cannot distinguish between opposing diagonals since the transform is still separable. To distinguish opposing diagonals with separable filters the filter frequency responses are required to be asymmetric for positive and negative frequencies. A good way to achieve this is to use complex wavelet filters which can be made to suppress negative frequency components. The Complex DWT has improved shift-invariance and directional selectivity than the separable DWT.

1. PRESENT THEORIES :

There has been a lot of research has been done in the field of image denoising. Fourier Transform along with its fast algorithms has been used for the analysis and processing of various signals. Fourier Transform has various limitations in the processing various signals, But a new tool Wavelet Transform has been emerged as a important method in analysis and processing of various signals [34]. With standard Discrete Wavelet Transforms, signal has a same data size in transform domain and therefore it is a non-redundant transform. Standard DWT can be implemented through a simple filter bank structure of recursive FIR filters. A very important property; Multiresolution Analysis (MRA) allows DWT to view and process different signals at various resolution levels. A wavelet is a small wave which has its energy concentrated in time. It has an oscillating wave like characteristic but also has the ability to allow

simultaneous time and frequency analysis and it is a suitable tool for transient, non-stationary or time-varying phenomena [33]. The Discrete Wavelet Transforms (DWT), obtained by iterating a perfect reconstruction (PR) filter bank (FB) on its low-pass output, decomposes a discrete-time signal according to octave-band frequency decomposition [1]. The new version of DWT, known as double-density DWT has the following important additional properties. (1) It employs one scaling function and two distinct wavelets which are designed to be offset from one another by one half. (2) The double density DWT is over complete by a factor of two [2].

Many different noise removal techniques have been applied to images, but the wavelet transform has been viewed by many as the preferred technique for noise removal. Rather than a complete transformation into the frequency domain, as in DCT (Discrete Cosine Transform)

or FFT (Fast Fourier Transform), the wavelet transform produces coefficient values which represent both time and frequency information. The hybrid spatial-frequency representation of the wavelet coefficients allows for analysis based on both spatial position and spatial frequency content. The hybrid analysis of the wavelet transform is excellent in facilitating image denoising algorithms [33]. Reference [4] illustrates the classification of the image denoising methods. The image denoising methods can be categorized according to estimation approach of wavelet coefficient and wavelet type. Table I shows classification according to wavelet coefficient. The Thresholding and Shrinkage methods give noticeable results. The related works and approaches are tabulated in below Table II.

The different types of wavelet and related works are tabulated in Table I.

Table I: Classification according to wavelet type

Wavelet Type	Related Works
Orthogonal separable Wavelet	Most of the papers
Translation Invariant Wavelet	[11,3,10,14]
Multi Wavelet	[16,3,10]
Complex wavelet	[8,18,33]

Table II Classification according to wavelet coefficient

Category	Approach	Related works
Thresholding	Universal thresholding	[27,7,12]
	Stein's Unbiased Risk Estimation (SURE)	[22]
	Cross validation	[25]
Shrinkage	Bayes Shrink	[15]
	MMSE	[14]
	Markov Random Field	[23]
	Adaptive Bayesian wavelet shrinkage (ABWS)	[9]
	Bivariate shrinkage using level dependency	[35]
	Neighbour dependency	[5,10]
Other	Hidden Markov Tree	[8]
	Gaussian scale mixture	[13]

2. BASICS TO THE DWT EXTENSIONS :

A filter bank plays an integral part in image denoising applications. The two filter banks namely, analysis filter bank and synthesis filter bank. A schematic representation of a filter bank is shown in Fig. 1 below. In Fig. 1, $x[n]$ is the input to the Analysis Filter Bank. The $Vo[n]$ is the output of the Analysis Filter bank. The output of the Synthesis Filter Bank is $y[n]$.

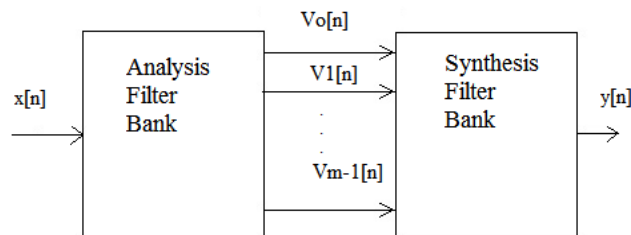


Figure 1:

Basic differences between the two DWT extensions:

The basic differences between the dual tree DWT and double density DWT are given below.

1] For the dual-tree DWT there are fewer degrees of freedom for design, while for the double-density DWT there are more degrees of freedom for design.

2] The dual-tree and double-density DWTs are implemented with totally different filter bank structures.

3] The dual-tree DWT can be interpreted as a complex-valued wavelet transform which is useful for signal modeling and denoising (the double-density DWT cannot be interpreted as such).

4] The dual-tree DWT can be used to implement two-dimensional transforms with directional wavelets, which is highly desirable for image processing [31].

By introducing this concept in discrete wavelet transform (DWT) we can achieve dual-tree DWT system. Also combining the double-density DWT and dual-tree DWT we can achieve the double-density dual-tree DWT system.

3. METHODOLOGY:

Double-Density Wavelet Transform:

The double-density DWT is an improvement upon the critically sampled DWT with important additional properties:

(1) It employs one scaling function and two distinct wavelets, which are designed to be offset from one another by one half,

(2) The double-density DWT is over complete by a factor of two

(3) It is nearly shift-invariant.

In two dimensions, this transform outperforms the standard DWT in terms of denoising; however, there is room for improvement because not all of the wavelets

are directional. That is, although the double-density DWT utilizes more wavelets, some lack a dominant spatial orientation, which prevents them from being able to isolate those directions.

A solution to this problem is provided by the double-density complex DWT, which combines the characteristics of the double-density DWT and the dual-tree DWT. The double-density complex DWT is based on two scaling functions and four distinct wavelets, each of which is specifically designed such that the two wavelets of the first pair are offset from one other by one half, and the other pair of wavelets form an approximate Hilbert transform pair. By ensuring these two properties, the double-density complex DWT possesses improved directional selectivity and can be used to implement complex and directional wavelet transforms in multiple dimensions.

We construct the filter bank structures for both the double-density DWT and the double-density complex DWT using finite impulse response (FIR) perfect reconstruction filter banks, which are discussed in detail at the beginning of each section. These filter banks are then applied recursively to the lowpass subband, using the analysis filters for the forward transform and the synthesis filters for the inverse transform. By doing this, it is then possible to evaluate each transform's performance in several applications including signal denoising, image enhancement.

Double-Density Discrete Wavelet Transform:

To implement the double-density DWT, we must first select an appropriate filter bank structure. The filter bank proposed in Figure 2 illustrates the basic design of the double-density DWT.

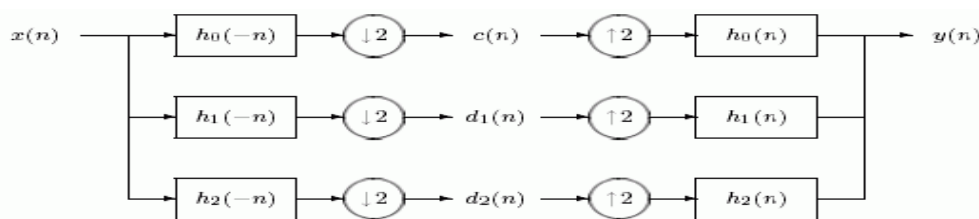


Figure 2: A 3-Channel Perfect Reconstruction Filter Bank.

The analysis filter bank consists of three analysis filters—one lowpass filter denoted by $h_0(-n)$ and two distinct highpass filters denoted by $h_1(-n)$ and $h_2(-n)$. As the input signal $x(n)$ travels through the system, the analysis filter bank decomposes it into three subbands, each of which is then down-sampled by 2. From this process we obtain the signals $c(n)$, $d_1(n)$, and $d_2(n)$, which represent the low

frequency (or coarse) subband, and the two high frequency (or detail) subbands, respectively.

The synthesis filter bank consists of three synthesis filters—one lowpass filter denoted by $h_0(n)$ and two distinct highpass filters denoted by $h_1(n)$ and $h_2(n)$ —which are essentially the inverse of the analysis filters. As the three subband signals travel through the system, they

are up-sampled by two, filtered, and then combined to form the output signal $y(n)$.

One of the main concerns in filter bank design is to ensure the perfect reconstruction (PR) condition. That is, to design $h_0(n)$, $h_1(n)$, and $h_2(n)$ such that $y(n)=x(n)$.

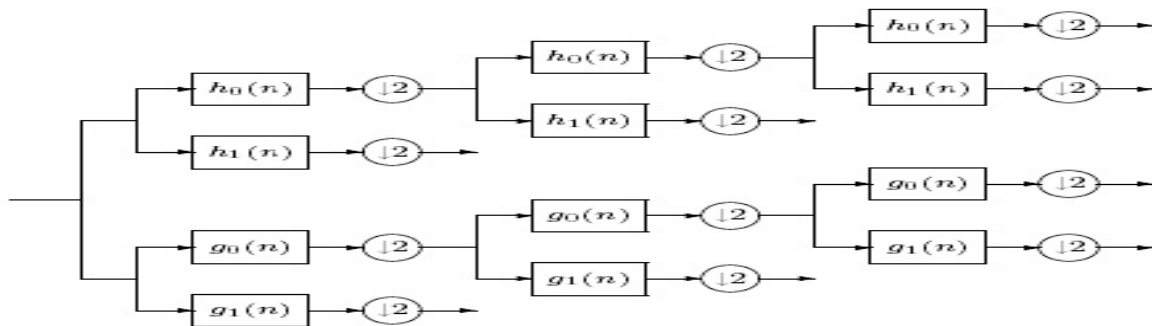


Figure 3:

The transform is 2-times expansive because for an N -point signal it gives $2N$ DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed in a specific way, then the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and subband signals of the lower DWT can be interpreted as the imaginary part. Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be an approximate Hilbert transform of the wavelet associated with the lower DWT. When designed in this way, the dual-tree complex DWT is nearly shift-invariant, in contrast with the critically-sampled DWT. Moreover, the dual-tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. (For the separable 2D DWT, recall that one of the three wavelets does not have a dominant orientation.) The dual-tree complex DWT outperforms the critically-sampled DWT for applications like image denoising and enhancement.

4. CONCLUSION:

In this paper we have discussed the various techniques of wavelet based denoising. The techniques used in image denoising are the extensions of the wavelet transform namely double density complex DWT.

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Dual-Tree Complex Wavelet Transform

The dual-tree complex DWT of a signal x is implemented using two critically-sampled DWTs in parallel on the same data, as shown in the figure 3.

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