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The fuzzy intuitive sets in the decision-making

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ABSTRACT

Every decision is possible with terms of psychology reducible to deciding between alternative main consideration and alternative options. The semantic information of intuitionistic fuzzy sets allows us to focus on the quantitative aspects of this type of decision-making that is based on only one main alternative. The article describes the method of estimation based on the fundamental uncertainties and makes quantify uncertainty and information that is needed for decision-making in such a simple model. From the described method is also possible to derive a generalization of complex decision-making situation. The environment of Matlab was used for modeling of relevant functions.

Key Words: Decision, Semantic information, Fuzzy set, Decision-making

INTRODUCTION:

The simplest model of decision-making assumes just one of two relevant variants. We are designate these considered alternatives by α, β. A certain degree of "power options" of some of variants is generally called uncertainty. Uncertainty as one of the characteristics of the decision-making process can be quantified on the basis of such a subjectively or objectively estimated probability $p \in \langle 0;1 \rangle$ as a choice of one of the two options, such as α, by the well-known Shannon´s Formula [1,2,3,4,6,10].

$$H(p) = -p \cdot \log_2 p - (1-p) \cdot \log_2(1-p)[bit], \tag{1}$$

where $H(p) \in \langle 0;1 \rangle$ with definition of $0 \cdot \log_2 0 = 0$.

One of the essential characteristics of the uncertainty (1) is its symmetry with respect to variants α, β:

$$H(p) = H(1-p);\ H(0{,}5) = 1 = max_p\{H(p)\};\ H(0) = H(1) = 0. \tag{2}$$

The quantity $I(p) = 1 - H(p)$ can be interpreted as a measure of information "consumed" by selecting one of the options. If we want the information to be semantized, i.e. polarized with respect to a certain variant, for example α, the semantized information $I^s(p)$ is chosen in the form [1, 2, 6]

$$I^s(p) = \begin{cases} -I(p); for\ 0 \le p \le 0{,}5 \\ I(p); for\ 0{,}5 < p \le 1 \end{cases} I^s(p) = -1 \le I^s(p) \le 1 = I^s(1);\ I^s(0{,}5) = 0. \tag{3}$$

1.UNCERTAINTY MODELLED WITH THE HELP OF FUZZY SETS:

Uncertainty may be estimated by using the probability study [5,7,8,9,11]. It is also possible to apply the decision-making model, characterized by the fuzzy set

$$\underline{A} = \{\alpha/\mu(\alpha);\ \beta/\mu(\beta)\}, \mu(\alpha) + \mu(\beta) \le 1, \tag{4}$$

where rate of plausibility $\mu(\alpha), \mu(\beta) \in \langle 0;1 \rangle$ are also subjective or objective estimated choice options. The uncertainty for fuzzy set (4) can be estimated by the following equation (5) with the choice of independent variations

$$\begin{aligned} H(\underline{A}) &= 2 - [max\{\mu(\alpha); 1-\mu(\alpha)\} + max\{\mu(\beta); 1-\mu(\beta)\}] = \\ &= min\{\mu(\alpha); 1-\mu(\alpha)\} + min\{\mu(\beta); 1-\mu(\beta)\} \end{aligned};\quad 0 \le H(\underline{A}) \le 1. \tag{5}$$

The information $I(\alpha)$ to select the variant α and its semantized form can be determined similarly as for $H(p)$ from following relationships:

$$I(\alpha) = 1 - H(\underline{A});\ I^s(\alpha) = \begin{cases} -I(\alpha); for\ 0 \le \mu(\alpha) \le 0{,}5 \\ I(\alpha); for\ 0{,}5 < \mu(\alpha) \le 1 \end{cases}. \tag{6}$$

Of course then it is $-1 \leq I^s(\alpha) \leq 1$. Negative information can be determined as a rate of disinformation or confusedness die to the choice of the variant α.

The essential characteristic of probability for the fuzzy set $\underline{A}$ need not to be generally applied

$$\mu(\alpha) + \mu(\beta) = 1. \tag{7}$$

However, that is because in our model choices are dependent on each other's variants, we assume the validity of equation (7). We only describe the situation of just one of the opinions. Then for uncertainty modelled by fuzzy (4) under condition (7) can be estimated, e.g. from the equation (8), which corresponds to (1):

$$H(\underline{A})(\alpha) = -\mu(\alpha) \cdot \log_2 \mu(\alpha) - \big(1 - \mu(\alpha)\big) \cdot \log_2\big(1 - \mu(\alpha)\big). \tag{8}$$

Then we also have $0 \leq H(\underline{A})(\alpha) \leq 1$, and so we can determine an information and its semantized shape as in the previous case. However, we have several options for estimating the uncertainty of $H(\underline{A})(\alpha)$ to meet the requirements of the intuitive conditions (2) and (7) [Půlpán, 2012]. For better understanding, we show another simple expression for the uncertainty $H(\underline{A})(\alpha)$, assuming the validity conditions (7). From the equation (5) we have

$$H(\underline{A})(\alpha) = 2 \cdot \min\{\mu(\alpha); 1 - \mu(\alpha)\};\ 0 \leq H(\underline{A})(\alpha) \leq 1. \tag{9}$$

To adapt the model of the simplest decision to the situation, where none of the options may be chosen in this process, it is reasonable to assume

$$\mu(\alpha) + \mu(\beta) \leq 1. \tag{10}$$

For example, when the decision-maker has a counter selecting bias towards some of the variants or one does not know how to choose. We can decide in this case to apply the use of intuitionist fuzzy set (IFS) $\mathcal{F}$ in order to characterize the decision-making as in the form [Atanassov, 1986]:

$$\mathcal{F} = \{\alpha/\big(\mu(\alpha);\ \nu(\alpha)\big);\ \beta/\big(\mu(\beta);\ \nu(\beta)\big)\}, \tag{11}$$

where $\mu(\alpha), \nu(\alpha) \in \langle 0; 1 \rangle$ and $\mu(\beta);\ \nu(\beta) \in \langle 0; 1 \rangle$ and apply (10) both α and β.

Deciding about the variant α and also according about the variant β is thus divided into three aspects:

1. Assessing the degree of acceptance of the variant α(estimated $\mu(\alpha)$)
2. Assessing the degree of disacceptance of the variant α(estimated $\mu(\alpha)$)
3. Assessing the degree of indecision for some variant of the α(defined by$1 - \mu(\alpha) - \nu(\alpha)$).

Corresponding fuzzy sets to the above types of decision are:

$\underline{A}_1(\alpha) = \{\alpha/\mu(\alpha);\ \neg\alpha/(1 - \mu(\alpha))\};\ \underline{A}_2(\alpha) = \{\alpha/\nu(\alpha);\ \neg\alpha/(1 - \nu(\alpha))\};\ \underline{A}_3(\alpha) = \{\alpha/\pi(\alpha);\ \neg\alpha/(1 - \pi(\alpha))\}$

where $\pi(\alpha) = 1 - \mu(\alpha) - \nu(\alpha)$ and their uncertainties $H\left(\underline{A}_1(\alpha)\right), H\left(\underline{A}_2(\alpha)\right), H\left(\underline{A}_3(\alpha)\right)$ determined according to (8) or (9). For ISF $\mathcal{F}$ from (11) can be introduced the uncertainty $H(\alpha)$. A variant of α regarding to the relationship

$$H(\alpha) = -(\mu(\alpha) \cdot \log_2 \mu(\alpha) + \nu(\alpha) \cdot \log_2 \nu(\alpha) + \pi(\alpha) \cdot \log_2 \pi(\alpha))(12)$$

and then

$$0 \leq H(\alpha) \leq \log_2 3. \tag{13}$$

The graph of uncertainty $H(\alpha)$we can see at figure (1).

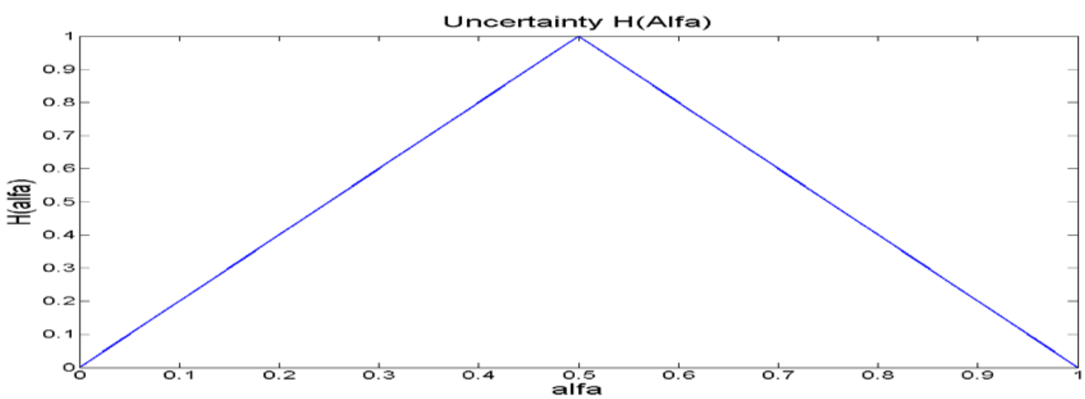

Figure 1: The graph of uncertainty $H(\alpha)$, see above (9).

For uncertainty $H(\alpha)$ and uncertainties $H\left(\underline{A}_1(\alpha)\right), H\left(\underline{A}_2(\alpha)\right), H\left(\underline{A}_3(\alpha)\right)$ are valid

$$H\left(\underline{A}_1(\alpha)\right) \leq H(\alpha); H\left(\underline{A}_2(\alpha)\right) \leq H(\alpha); H\left(\underline{A}_3(\alpha)\right) \leq H(\alpha). \quad (14)$$

This follows from the following sequence of inequalities:
$\log_2(\nu+\pi) \geq \log_2 \nu;\ -\log_2(\nu+\pi) \leq -\log_2 \nu;\ -\log_2(\nu+\pi) \leq -\log_2 \nu;\ etc.$

Inequality (13) gives us the opportunity to determine information due to the variant α for IFS $\mathcal{F}$ information $I_\alpha(\mathcal{F})$ in equation (15a)

$$I_\alpha(\mathcal{F}) = \log_2 3 - H(\alpha) (15a)$$

or based on its normalized form

$$I_\alpha(\mathcal{F})_{norm} = 1 - H(\alpha)/\log_2 3. \quad (15b)$$

Corresponding relationship for the semantisation

$$I_\alpha(\mathcal{F})_{norm}{}^S = \begin{cases} -I_\alpha(\mathcal{F})_{norm}; 0 \leq \mu(\alpha) \leq 0{,}5 \\ I_\alpha(\mathcal{F})_{norm}; 0{,}5 \leq \mu(\alpha) \leq 1 \end{cases}, \quad (16)$$

where $-j \leq I_\alpha(\mathcal{F})_{norm}{}^S \leq 1$.

When selecting just one of the possible variants of α, β in our model of decision making described by IFS $\mathcal{F}$, we must also apply initial conditions (7), then must be

$$\nu(\alpha) + \nu(\beta) \leq 1. \quad (17)$$

Uncertainty for IFS can be defined under all these conditions by the relationship

$$H(\mathcal{F})(\alpha) = H\left(\underline{A}_1(\alpha)\right) + H\left(\underline{A}_2(\alpha)\right) + H\left(\underline{A}_3(\alpha)\right) (18)$$

and $0 \leq H(\mathcal{F})(\alpha) \leq 3$. Therefore we can define information for IFS $\mathcal{F}$ and also information $I(\mathcal{F})(\alpha)$ by relationship $I(\mathcal{F})(\alpha) = 3 - H(\mathcal{F})(\alpha)$ or as a standardized information $I_\alpha(\mathcal{F})_{norm} = 1 - H(\mathcal{F})(\alpha)/3$. We can also transfer the given standardized information corresponding to a semantic appearance according to a variant α, just as we did in the previous cases. We choose

$$I(\mathcal{F})^S(\alpha) = \begin{cases} -I(\mathcal{F})_{norm}; 0 \leq \mu(\alpha) \leq 0{,}5 \\ I(\mathcal{F})_{norm}; 0{,}5 \leq \mu(\alpha) \leq 1 \end{cases}, \quad -1 \leq I(\mathcal{F})^S(\alpha) \leq 1. \quad (19)$$

The graph of semantized information we can see at figure (2).

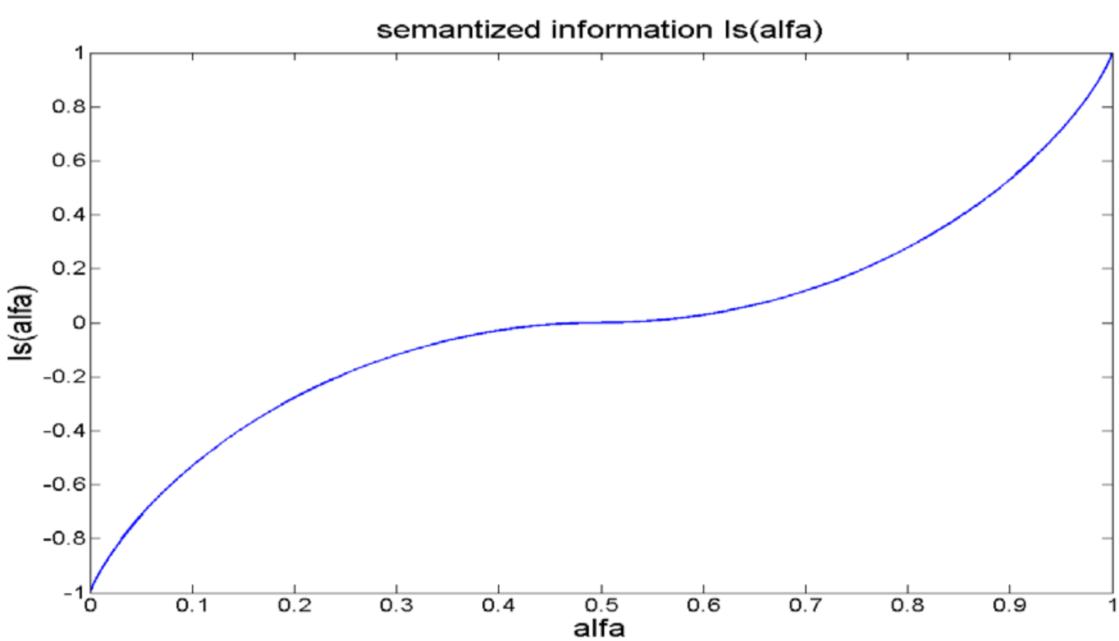

Figure 2: ***The graph of semantized information $I(\mathcal{F})^S(\alpha)$, see above (19).***

2.EXAMPLE:

The patient arrived the doctor, which suffered from itching the upper and lower limbs of skin. There were a places sharply bounded intense red coloration on the skin. Initially the doctor decided the patient suffer from dermatitis and began to treat the patient by applying an ointment to affected areas. Nevertheless, the patient's troubles did not improve after a few weeks. Red discoloration of the skin was permanently expanding and itching persisted. The doctor changed his mind on the diagnosis, considering the disease is psoriasis. The biopsy of patient's affected skin was done for confirming or excluding of suspicion. At the same time, the patient was sent to have an allergy and internal examination. Examination did not confirming the disease from psoriasis, but unequivocally it did not exclude. Although the ointments were changed further treatment was still unsuccessful. That was the reason for realizing the medical counseling where the following treatment should be discuss. Doctors agreed that there should be mainly excluded a disease from psoriasis. Due to further examination and a new hypotheses about the cause of the disease could be assumed variants of the disease less important.

Let us now formally mathematically express the described uncertainty of described stages of thinking about possible patient's disease using intuitive fuzzy sets. Denote therefore hypotheses and their rate of credibility, estimated from the opinions of a group of doctors from counselling as follows:

α - the patient became ill with psoriasis, the estimates are $\mu(\alpha), \nu(\alpha)$

β_1 - the patient became ill with dermatitis, the estimates are $\mu(\beta_1), \nu(\beta_1)$

β_2 - the patient had an allergic reaction, the estimates are $\mu(\beta_2), \nu(\beta_2)$

β_3 - skin lesions are psychological in origin, the estimates are $\mu(\beta_3), \nu(\beta_3)$

Because the doctors came to an agreement that should be mainly excluded disease psoriasis, the estimate of uncertainty is realized due to α. Therefore we denote $\beta = \beta_1 \vee \beta_2 \vee \beta_3$.

Informed doctors estimated for α: $\mu(\alpha) = 0{,}3;\ \nu(\alpha) = 0{,}6$; then $\pi(\alpha) = 1 - \mu(\alpha) - \nu(\alpha) = 0{,}1$. According (8) we have in units bit: $H(A_1(\alpha)) = 0{,}881$; $H(A_2(\alpha)) = 0{,}971$; $H(A_3(\alpha)) = 0{,}469$.

According (12) we get in units bit: $H(\alpha) = 0{,}521 + 0{,}442 + 0{,}332$.

According (15b) we compute in units bit: $I_\alpha(\mathcal{F})_{\text{norm}} = 0{,}183$and according (16) is $I_\alpha(\mathcal{F})_{\text{norm}^s} = -0{,}183$.

From (18) we obtain by calculation $H(\mathcal{F})(\alpha) = 2{,}321$, $I(\mathcal{F})(\alpha) = 0{,}679$, $I(\mathcal{F})_{norm}(\alpha) = 0{,}226$, all in bit units.

From equation (19) we determine $I(\mathcal{F})^s(\alpha) = -0{,}226$ [bit].

We can similarly determine values of uncertainty from (9) in lit units: $H(A_1(\alpha)) = 0{,}6$; $H(A_2(\alpha)) = 0{,}8$; $H(A_3(\alpha)) = 0{,}2$. According (18) we get $H(\mathcal{F})(\alpha) = 1{,}6$ and $I(\mathcal{F})_{norm}(\alpha) = 0{,}47$ and according (19) we get $I(\mathcal{F})^s(\alpha) = -0{,}47$.

In our investigation, it was possible to estimate an appropriate rate $\mu(\beta), \nu(\beta)$ for alternate β, which is an alternative to the variant α so that we could determine intuitive fuzzy set (IFS) $\mathcal{F}$. The result of estimation was $\mathcal{F}$=$\{\alpha/(0{,}3;\ 0{,}6); \beta/(0{,}7;\ 0{,}2)\}$. Because the (7) is valid there, we can calculate the uncertainty of the diagnosis α in respect of diagnosis from β admit (8) by the relationship

$$H(\mathcal{F}, \alpha) = -(\mu(\alpha)\log_2 \mu(\alpha) + \mu(\beta)\log_2 \mu(\beta)) = 0{,}521 + 0{,}36 = 0{,}881\ [bit].$$

Relevant semantized information with respect to α then is $I^s(\mathcal{F}, \alpha) = -0{,}119\ [bit]$. When we proceeded semanticisation we already have used the known way. We get $H(\mathcal{F}, \alpha) = 0{,}6\ [lit]$ by calculation with respect to (9). Relevant semantized value of information with respect to α is then $I^s(\mathcal{F}, \alpha) = -0{,}4\ [lit]$.

Another question is how the variability in estimating the appropriate credibility may affect the values of semantized information. We estimate therefore a mistake for uncertainty $H(\mathcal{F}, \alpha)$, when $\mu(\alpha) = \bar{\mu}(\alpha) \pm \Delta\mu(\alpha)$, where $\bar{\mu}(\alpha)$ is a point estimate of the value $\mu(\alpha)$ from the measurement (e.g. arithmetic mean from the experimental data) and $\Delta\mu(\alpha)$ is an estimate of the measurement error (e.g. standard deviation of measurements):

$$\Delta H(\mathcal{F}, \alpha) \sim \frac{dH}{d\mu} \cdot \Delta\mu(\alpha) = \left(\log_2 \frac{1-\mu}{\mu}\right) \cdot \Delta\mu(\alpha).$$

We assume the error of estimating $\mu(\alpha)$ is equal $0{,}1$. Then we establish when $\mu(\alpha) = 0{,}3 \pm 0{,}1$ the error of estimation $H(\mathcal{F}, \alpha)$ by value

$$H(\mathcal{F},\alpha) = \log_2 \frac{1-0{,}3}{0{,}3} \cdot 0{,}1 = 0{,}122\ [bit].$$

Therefore, we can write $H(\mathcal{F},\alpha) = 0{,}226 \pm 0{,}122\ [bit]$. The same error of $\pm 0{,}122$ has the relevant semantized information $I^s(\mathcal{F},\alpha)$. As an information gain of the diagnosis of psoriasis in these conditions can be considered the values another semantized information with the relevant interpretations, especially value of $I^s(\mathcal{F},\alpha)$. For choice of diagnosis of psoriasis we calculate a small value of semantized information that moves around zero on scale $\langle -1; 1 \rangle$.

3. Conclusion

Values of semantized information are determined depending on the choice of computational methods and units. Interpretation of the calculated numerical values is then given the underlying assumptions of method and experimenter experience in a particular field of diagnoses or scientific decisions. This method is more suited to comparing values of semantized information for different diagnosis or scientific cases. It also depends on the experience of the experimenter, which method of calculation he will vote and how he learn to interpret the numerical values.

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